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CHARACTERIZING THE DIVIDED DIFFERENCE WEIGHTS FOR EXTENDED COMPLETE TCHEBYCHEFF SYSTEMS

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Newman and Rivlin have shown that there is a 1-1 correspondence between the nodes and weights of the nth order divided difference of nth degree polynomials. Their method applies only to polynomials. In this paper we develop a new approach and apply it to extend their results to the setting of Extended Complete Tchebycheff Systems.

0. Introduction. In [7] Newman and Rivlin (see also the reference there to S. Karlin's results) were able to characterize the weights which appear in the nth order divided difference formula with respect to the base functions $\{u_j(x) = x^j\}_{j=0}^n$ and to establish a 1-1 correspondence between these weights and the corresponding set of nodes, $0 = x_0 < x_1 < \cdots < x_n$, used in the formula. We propose in this paper to extend this result to the setting where the family $\{u_j(x)\}_{j=0}^n$ forms an Extended Complete Tchebycheff System (E.C.T.S.) on $[0, \infty)$. This means for each k, where $0 \le k \le n$, any non-trivial linear combination of the functions $\{u_0, \ldots, u_k\}$ has at most k zeros (including multiplicities) in $[0, \infty)$ where each $u_j \in C^n[0, \infty)$. We further assume that $u_0(x) \equiv 1$. For the remainder of this paper we shall postulate that these basic hypotheses concerning $\{u_j\}_{j=0}^n$ hold.

Among the E.C.T.S. for which these results are valid, we will highlight the families generated by the Cauchy Kernel and the Exponential Kernel.

1. Statement of problem. Let

$$(1) \qquad S = \{\mathbf{x} = (x_1, \ldots, x_n) \subset R^n \colon 0 < x_1 < \cdots < x_n\}, \qquad x_0 \equiv 0.$$

A is defined to be the set of all $\mathbf{a} = (a_0, \ldots, a_n) \in R^{n+1}$ such that the following properties are valid

$$(2) \qquad \begin{aligned} &\text{(i)} \quad (-1)^{n-i} a_i > 0 \qquad (i = 0, 1, \ldots, n); \\ &\text{(ii)} \quad \sum_{i=0}^{n} a_i = 0; \\ &\text{(iii)} \quad (-1)^{n-j} \sum_{i=j}^{n} a_i > 0, \quad j = 1, \ldots, n. \end{aligned}$$