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FLAT HILBERT CUBE MANIFOLD PAIRS

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The purpose of this paper is to study embeddings of Q-manifolds into Q-manifolds. Mainly, we relate flat Q-manifold pairs with PL manifold pairs by using a relative version of the Chapman Splitting Theorem. The concepts of QPL embedding and QPL homeomorphism are introduced.

1. Introduction and definitions. For topological spaces (polyhedra) X and Y an embedding $f\colon X \to Y$ is said to be a (PL) locally flat embedding provided that every point of X has a neighborhood U and an open (PL) embedding $h\colon U \times \mathbf{R}^m \to Y$ such that $h(x,0) = f(x)$, for all $x \in U$. If U can be taken to be all of X, then the embedding is said to be a (PL) flat embedding. Furthermore, the pair (Y, X) is said to be a flat pair if the inclusion $X \hookrightarrow Y$ is a (PL) flat embedding. Note that if (M, N) is a flat finite-dimensional manifold pair, then $N \cap \partial M = \partial N$ and $(\partial M, \partial N)$ is a flat manifold pair.

We use Q to denote the Hilbert cube and by a Q-manifold we mean a separable metric manifold modeled on Q.

The purpose of this paper is to relate flat Q-manifold pairs with flat PL manifold pairs by using a relative version of the Chapman Splitting Theorem [**6**]. The following is our first result in this direction.

THEOREM 1. *Let $(\mathfrak{M}, \mathfrak{N})$ be a flat compact Q-manifold pair. Then there exists a flat PL manifold pair (M, N) and a homeomorphism h: $(\mathfrak{M}, \mathfrak{N}) \to (M, N) \times Q$.*

Chapman [**4**] has proved that there exists a codimension 3 locally flat embedding $\mathfrak{N} \hookrightarrow \mathfrak{M}$ between Q-manifolds such that $\mathfrak{N}$ has no tubular neighborhood and, moreover, no stabilization $\mathfrak{N} \times \{0\} \hookrightarrow \mathfrak{M} \times \mathbf{R}^n$ has a tubular neighborhood. On the other hand, Milnor [**9**] and Kister [**8**] proved the stable existence of tubular neighborhoods for embeddings of finite dimensional manifolds. Consequently an analogue of Theorem 1 for locally flat Q-manifold pairs is not possible.

Let M and N be PL manifolds. An embedding (homeomorphism) f: $N \times Q \to M \times Q$ is said to be a QPL embedding (homeomorphism) if there exists a PL embedding (homeomorphism) g: $N \times I^n \to M \times I^m$