PACIFIC JOURNAL OF MATHEMATICS
Vol. 112, No 2, 1984

DOUBLE BRANCHED COVERS AND PRETZEL KNOTS

RICHARD E. BEDIENT

Given a knot K we describe a modification of K which leaves the double branched cover of S^3 branched along K unchanged. We then modify certain pretzel knots in this way to produce arbitrarily large families of distinct knots having the property that all of the associated double branched covers are homeomorphic.

1. Introduction. This paper concerns the relationship between a knot and its associated double branched cover. A brief review of the history of this problem will indicate some of the known results.

Given a tame knot K in S^3 the unique representation of $\Pi_1(S^3 - K)$ onto Z_2 yields a unique closed orientable 3-manifold $M(K)$ called the double branched cover of S^3 branched along K. By Waldhausen [**17**] K is trivial if and only if $M(K) \approx S^3$. This leads one to ask whether knot types and the homeomorphism types of their branched covers are in one-to-one correspondence. Birman and Hilden [**2**] give an affirmative answer in the case where the Heegaard genus of $M(K)$ is 2 and where homeomorphism type is replaced by Heegaard splitting class.

Unfortunately (or possibly fortunately) when the above restriction on genus is removed, counter-examples appear in abundance. Montesinos [**8**] and Viro [**16**] independently give examples of distinct composite links for which the double covers are homeomorphic. Birman, Gonzalez-Acuna and Montesinos [**3**] remove the restrictions "composite" and "link" by producing pairs of distinct prime knots such that for each pair the double covers are the same. These examples are also described in different ways by Takahashi [**14**] and Bedient [**1**]. Montesinos [**7**] has also given examples of arbitrarily large families of distinct *links* such that within each family all of the associated double covers are homeomorphic.

In this paper we will show that such families of *knots* exist. Boileau and Siebenmann [**4**] have obtained similar examples.

This result can be interpreted in a different manner as follows. We note that the manifold constructed is a Seifert fibered manifold which then admits n distinct involutions where distinct here means non-conjugate in the automorphism group. For more on this see Plotnick [**11**].