

SHALIKA'S GERMS FOR p -ADIC $GL(n)$, II: THE SUBREGULAR TERM

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For an elliptic torus in $GL(n)$ over a p -adic field an explicit formula is established for the germ associated to the "subregular" unipotent class, i.e. the class whose Jordan canonical form contains a 1×1 block and an $(n - 1) \times (n - 1)$ block. In particular this, together with previously known information, gives all the germs for $GL(3)$.

0. In [1], an ad hoc method was described for calculating the germ associated to the regular unipotent class. Here that approach is refined to deal with the subregular class. Neither the technique nor the final result is particularly clean; it would be desirable to express the germ in terms more suggestive of generalizations.

The results obtained here are consistent with conjectures made by J. Rogawski in his thesis ([3]).

The idea is to construct a function f whose orbital integrals vanish for all unipotent classes except the regular and subregular classes. The germ can easily be calculated from the unipotent orbital integrals and the orbital integrals of f over the classes of regular elements of an elliptic torus.

§1 establishes notations and defines the function f . §§2–7 contain the calculation of the elliptic orbital integrals of f , which are given by Proposition 4. The unipotent orbital integrals are given in §8, and the Theorem in §9 contains the main result, a formula for the subregular germ, preceded by a brief summary of the notation. Finally, §10 describes the result for $GL(3)$ more explicitly.

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1. Let F be a p -adic field, $\mathfrak{o} = \mathfrak{o}_F$ and $\mathfrak{p} = \mathfrak{p}_F$ its ring of integers and prime ideal, respectively, and $q = |\mathfrak{o}/\mathfrak{p}|$. Let $G = GL(n, F)$, $K = GL(n, \mathfrak{o})$, and $K_1 = \{k \in K: k \equiv \text{id}, \text{ mod } \mathfrak{p}\}$, the congruence subgroup.