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CLASSIFICATION OF ALGEBRAIC SURFACES WITH SECTIONAL GENUS LESS THAN OR EQUAL TO SIX. I: RATIONAL SURFACES

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Using Sommese's results on the adjunction process we give a biholomorphic classification of rational algebraic surfaces with the genus of a hyperplane section less than or equal to six.

Introduction. In this paper we have given a biholomorphic classification of smooth, connected, projective, rational surfaces X with smooth hyperplane section C such that the genus g of C is less than or equal to six. In 1936, L. Roth in [**R**] had given a birational classification of such algebraic surfaces with g less than or equal to six. Our biholomorphic classification goes considerably beyond his classification. Let $L = [C]$ for some hyperplane section C. Since in the cases in which $g = 0$, 1, X has been completely classified by Del Pezzo [**N**], we need only to consider the cases in which $g = 2,\ldots,6$. Since the cases $g = 2$, 3, 4 follow very easily from A. J. Sommese [**So**], the really interesting cases are $g = 5$, 6. Our classification has a slight overlap with P. Ionescu [**Io**]. Our classification is essentially based on the adjunction process which was introduced by the Italian school and which has been particularly studied by Sommese [**So**]. Our notations are as in [**So**] except for the following. X will denote a smooth, connected, projective, rational surface and L a very ample line bundle on X. Let $\bar{L} = K_X \otimes L$. Then $\phi_{\bar{L}}$ is the map given by the sections of the line bundle $\bar{L}$. Sommese, [**So**, (2.0.1) pg 390], has proved that $\dim \phi_{\bar{L}}(X) = 0$ if and only if $g = 1$. If X is rational then $\dim \phi_{\bar{L}}(X) = -\infty$ if and only if $h^{1,0}(X) = \dim H^1(X, \mathcal{O}_X) = 0$ and in this case (X, L) has been classified by Del Pezzo, (0.6). Let $\phi_{\bar{L}} = r \circ s$ be the Remmert-Stein factorization of $\phi_{\bar{L}}$. In the case in which $\dim \phi_{\bar{L}}(X) = 1$, it follows from Sommese [**So**, (2.1.1) pg 390] that, since $h^{1,0}(X) = 0$ in our case, s is an embedding. If $\dim \phi_{\bar{L}}(X) = 2$, Sommese [**So**, (2.3) pg 392], has proved that there exists a pair $(\hat{X}, \hat{L})$ such that:

(a) X is obtained by blowing up a finite set of points F of $\hat{X}$, π: $X \to \hat{X}$.

(b) Every smooth hyperplane section $C \in |L|$ is the proper transform of a smooth hyperplane section $\hat{C} \in |\hat{L} - F|$.