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# DESUSPENSIONS OF STUNTED PROJECTIVE SPACES

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**For multiples of the Hopf bundles over real and complex projective spaces, the number of sections is compared with the number of times that the Thom complex desuspends. Examples of equality of these numbers, and of inequality, are given.**

**1. Survey of Results.** For $F$ the field of real numbers $R$ or complex numbers $C$, let $FP_m^{m+n}$ denote the stunted $F$-projective space $FP^{m+n}/FP^{m-1}$. In this paper, which combines some new results with a survey of known results, we consider the question of finding the largest $s$ such that $FP_m^{m+n}$ desuspends $s$ times, i.e. for which there exists a complex $Y$, often denoted $\Sigma^{-s}FP_m^{m+n}$, such that $\Sigma^s Y$ and $FP_m^{m+n}$ have the same homotopy type. This question is a good test for virtually any technique in (unstable) homotopy theory; e.g. secondary cohomology operations ([**4**]), Maunder operations ([**5**]), Adams operations in loop spaces ([**12**]), and unstable $BP$-operations ([**36**]). It is of particular importance because of its relation to the generalized vector field problem; since $FP_m^{m+n}$ is the Thom complex of $m\xi_{F,n}$, where $\xi_{F,n}$ denotes the Hopf bundle over $FP^n$, the following proposition is immediate (and very well known).

PROPOSITION 1.1. *If $m\xi_{F,n}$ has $s$ sections, then $\Sigma^{-s}FP_m^{m+n}$ exists.*

These sectioning results are often conveniently expressed in terms of geometric dimension (gd), for which the desuspension analogue is the number $b(FP_m^{m+n})$, where $b(X)$ is defined in §2. It is the minimal dimension of bottom cells of iterated desuspensions of iterated suspensons of $X$. We will show in 2.4 that for $X = FP_m^{m+m}$ the iterated suspensions can be omitted. Then 1.1 becomes

PROPOSITION 1.1′. $b(FP_m^{m+n}) \le \mathrm{gd}(m\xi_{F,n})$.

The major theme of this paper will be to assess how strong 1.1 is, i.e. to determine which of the known nonsectioning results are implied by nondesuspensions.