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ON THE LIFTING THEORY OF FINITE GROUPS OF LIE TYPE

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Let G be a connected reductive algebraic group defined over a finite field $\mathbf{F}_q$ of characteristic $p > 0$, $q = p^a$. Let F be a corresponding Frobenius endomorphism such that $\mathbf{G}^{F^m} = \{g \in G\colon F^m(g) = g\}$ is a finite group of Lie type for a positive integer m. In this paper we discuss various aspects of the lifting theory of these finite groups.

0. Introduction. The paper is divided into four sections. In §1. N. Kawanaka's norm map is defined and admissible integers are discussed. In §2 the lifting theory of $G_2(q)$ is described. §3 is devoted to liftings of certain principal series representations of groups of adjoint type. Finally, we prove (in §4) that the duality operation defined by C. W. Curtis [7] commutes with lifting.

We use the notation $\mathbf{G}^{F^m} = G(q^m)$, $\operatorname{Irr} H =$ set of irreducible characters of a finite group H, $\overline{\mathbf{F}}_q =$ algebraic closure of $\mathbf{F}_q$, and $A = \langle F|_{G(q^m)} \rangle$. A acts on $G(q^m)$, and is a cyclic group of order m. Embed A and $G(q^m)$ into the semidirect product $A \cdot G(q^m)$. If $\chi \in \operatorname{Irr} G(q^m)$ is F-invariant, it extends to $\chi' \in \operatorname{Irr} A \cdot G(q^m)$. There is a norm map $\mathfrak{N}$ which yields a bijection $\{A \cdot G(q^m)\text{-conjugacy classes in } F \cdot G(q^m)\} \leftrightarrow \{\text{conjugacy classes of } G(q)\}$ (see §1).

DEFINITION. Let $\theta \in \operatorname{Irr} G(q)$. Then $\psi \in (\operatorname{Irr} G(q^m))^F$ is the *lift* of θ if ψ extends to $\psi' \in \operatorname{Irr} A \cdot G(q^m)$ and satisfies $\psi'(Fy) = C\theta(\mathfrak{N}(y))$ for some constant C and for all $y \in G(q^m)$.

In 1976 a paper of T. Shintani [16] was published which described the lifting theory of the finite groups $\mathrm{GL}(n, q)$. This marked the beginning of the lifting theory of finite groups of Lie type. Kawanaka subsequently developed much of the theory in his papers on $\mathrm{U}(n, q)$ [11] and on $\mathrm{Sp}(2n, q)$, $\mathrm{SO}(2n + 1, q)$, and $\mathrm{SO}^{\pm}(2n, q)$ [12]. We will consider the finite exceptional groups other than $G_2(q)$ in future papers (work is in progress).

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