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ORDINARY AND SUPERSINGULAR COVERS IN CHARACTERISTIC p

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This paper studies Galois wildly ramified covers of the projective line in characteristic p. It is shown that for p-covers of tamely ramified covers, the monodromy is "generated by the branch cycles." But examples are given to show that this condition fails in general for towers taken in the opposite order and for other covers as well—even in the case of covers branched only over infinity. It is also shown that p-covers branched at a single point are supersingular and more generally that for any curve which arises as a p-cover, there is a bound on the p-rank which in general is less than the genus.

In 1957, S. Abhyankar observed [**Ab**] that while the monodromy group of a branched covering of the Riemann sphere is generated by loops around the branch points, the analogous condition fails to hold in characteristic p. He conjectured that the condition at least holds for tamely ramified covers. This is indeed the case, as A. Grothendieck showed by the technique of specialization (XIII, Cor. 2.12 of [**Gr**]). In §1 of this paper, we show that it also holds for Galois covers which are the "opposite" of tame—viz. those whose Galois group is a p-group. More generally, we show that Galois covers which arise as p-covers of tamely ramified covers are "ordinary" (i.e. satisfy the above condition). But as §1 shows, towers taken in the opposite order need not satisfy this condition, nor does every "extraordinary" cover arise in this manner. We also discuss the connection to the problem of groups occurring as Galois groups over the affine line. Section 2 relates these ideas to supersingularity, and more generally to the phenomenon of a curve having fewer étale p-covers than "expected" for its genus. It is shown that an ordinary cover of the projective line which is branched over a single point must be supersingular. More generally, a bound is given on the number of étale $\mathbf{Z}/p$-covers of a curve which arises as a branched p-cover of another curve, in terms of the degree and the ramification groups.

We fix our terminology: All curves are assumed to be smooth, and defined over an algebraically closed field k. If X is a connected curve, then a (branched) cover $Z \to X$ is a morphism of curves which is finite and generically separable. The branch locus is thus finite, and $Z \to X$ is étale if the branch locus is empty. A cover $Z \to X$ is called *Galois* with group G if