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ON THE DWORK TRACE FORMULA

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We prove a generalization of Dwork's trace formula for certain completely continuous operators on p-adic Banach spaces. This generalization makes it simpler to apply Dwork's theory to the study of certain exponential sums involving both additive and multiplicative characters. As an example, we treat the case of Gauss sums and give a new proof of the Gross-Koblitz formula.

0. Introduction. The Dwork Trace Formula is a basic tool for applying the techniques of p-adic analysis to the study of exponential sums with an additive character. Let p be a prime and let $\mathbf{F}_q$ be a finite field with $q = p^f$ elements. Let $\Psi: \mathbf{F}_q \to \mathbf{C}^\times$ be an additive character. For $f \in \mathbf{F}_q[x_1,\ldots,x_n]$, define an exponential sum

$$S(f) = \sum_{x_1,\ldots,x_n \in \mathbf{F}_q} \Psi(f(x_1,\ldots,x_n)). \tag{0.1}$$

Bombieri [1] has used the Dwork Trace Formula to study such exponential sums and their associated L-functions. The purpose of this article is to prove a generalization of the Dwork Trace Formula (Theorem 1) which will allow one to treat in a straightforward manner sums of the form

$$\sum_{x_1,\ldots,x_n \in \mathbf{F}_q} \chi_1(x_1) \cdots \chi_n(x_n) \Psi(f(x_1,\ldots,x_n)), \tag{0.2}$$

where $\chi_1,\ldots,\chi_n: \mathbf{F}_q^\times \to \mathbf{C}^\times$ are multiplicative characters. Such sums can be handled by the earlier trace formula at the expense of certain technical complications, i.e., change of variable in the polynomial f, which results in changes in the Frobenius operator and the differential operators with which Frobenius commutes (see for example [4, eqs. (6.47), (6.48), and (6.49)]). Our point here is that by enlarging the space on which Frobenius operates, one obtains the sums (0.1) and (0.2) from the same Frobenius operator, hence the commuting differential operators are unchanged also. This enables one to apply the other elements of Dwork's theory more directly.

As an example, in §2 we give another proof of the Gross-Koblitz formula. We follow the ideas of [2], although we simplify by avoiding any appeal to the dual theory. We hope that the ideas of this paper will lead to