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A SOLUTION TO A PROBLEM OF E. MICHAEL

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A product space $X \times Y$ is *rectangularly normal* if every continuous real-valued function defined on a closed rectangle $A \times B$ in $X \times Y$ can be continuously extended onto $X \times Y$. It is known that products of normal spaces with locally compact metric spaces are rectangularly normal. In this paper we prove the converse of this theorem by showing there exists a normal space X such that its product $X \times M$ with a metric space M is rectangularly normal if and only if M is locally compact, thus answering positively a question raised by E. Michael.

Other related results are obtained; in particular, we show there exists a normal space X and a countable metric space M with one non-isolated point such that the product space $X \times M$ is not *rectangular* (in the sense of Pasynkov).

1. Introduction. Let R, Q and I denote the reals, the rationals and the unit segment. We say that a product space $X \times Y$ is *rectangularly normal* if every continuous real-valued function $f\colon A \times B \to R$ defined on a closed rectangle $A \times B$ in $X \times Y$ can be continuously extended onto $X \times Y$. The concept of rectangular normality—being a natural weakening of normality—first appeared implicitly in papers of Morita [**M9**], Starbird [**S, S2**] and Miednikov [**Mi**] in connection with their successful attempt to generalize the Borsuk Homotopy Extension Theorem. It turned out that even though normality and countable paracompactness of X are necessary (and sufficient) for the normality of the product $X \times I$, only normality of X suffices to ensure rectangular normality of $X \times I$. More generally, the following theorem holds:

1.1. THEOREM [**Mo, S2, Mi**]. *Products of normal spaces with locally compact metric spaces are rectangularly normal.* □

In this paper we prove the converse of this Theorem by showing that there exists a normal space X whose product with a metric space M is rectangularly normal if and only if M is locally compact (Example 2.5). In particular, X is a normal space whose product, $X \times Q$, with the space of rationals Q is not rectangularly normal. This answers a question raised by E. Michael.

The existence of the above space X is a consequence of Theorem 2.4, which states that $X \times M$ is rectangularly normal for some non-locally compact metric space M if and only if X is countably functionally Katětov