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A SPECTRAL DUALITY THEOREM FOR CLOSED OPERATORS

I. ERDELYI AND WANG SHENGWANG

The spectral duality theorem asserts that a densely defined closed operator T induces a spectral decomposition of the underlying Banach space X iff the conjugate T^* induces the same type of spectral decomposition of the dual space X^*. This theorem is known for bounded linear operators in terms of residual (S)-decomposability. In this paper we extend the spectral duality theorem to unbounded operators, under a general type of spectral decomposition. Our approach to the spectral duality leads through the successive conjugates T^*, T^{**} and T^{***} of T, under their domain-density assumptions.

1. Elements of local spectral theory for a closed operator. X is an abstract Banach space over the complex field $\mathbf{C}$. If S is a set, we write $\bar{S}$ for the closure, $\operatorname{Int} S$ for the interior, S^c for the complement, ∂S for the boundary, and $\operatorname{cov} S$ for the collection of all finite open covers of S. If S is a subset of $\mathbf{C}$, then the above mentioned topological constructs are referred to the topology of $\mathbf{C}$. Without loss of generality, we assume that for $S \subset \mathbf{C}$, each $\{G_i\}_{i=0}^n \in \operatorname{cov} S$ has, at most, one unbounded set G_0. An open $G \subset \mathbf{C}$ is said to be a neighborhood of ∞, in symbols $G \in V_\infty$, if for $r > 0$ sufficiently large, $\{\lambda \in \mathbf{C}: |\lambda| > r\} \subset G$. We write $S^\perp$ for the annihilator of $S \subset X$ in X^* (as well as that of $S \subset X^{**}$ in X^{***}) and ${}^\perp S$ for the preannihilator of $S \subset X^*$ in X (or that of $S \subset X^{***}$ in X^{**}). $B(X)$ denotes the Banach algebra of all bounded linear operators which map X into X. I stands for the identity operator.

For a linear operator T: D_T $(\subset X) \to X$, we use the following notations: spectrum $\sigma(T)$, resolvent set $\rho(T)$, and resolvent operator $R(\cdot\,; T)$.

If T has the single valued extension property (SVEP) then, for $x \in X$, $\sigma_T(x)$ is the local spectrum, $\rho_T(x)$ is the local resolvent set and $x(\cdot)$ is the local resolvent function. For $S \subset \mathbf{C}$, an extensive use will be made of the spectral manifold $X(T, S) = \{x \in X: \sigma_T(x) \subset S\}$.

$\operatorname{Inv} T$ represents the lattice of all invariant subspaces under T. For $Y \in \operatorname{Inv} T$, $T|Y$ is the restriction of T to Y and T/Y denotes the coinduced operator on the quotient space X/Y with domain $D_{T/Y} = \{\hat{x} \in X/Y: \hat{x} \cap D_T \neq \varnothing\}$.

If not mentioned otherwise, throughout this paper T is a densely defined unbounded closed operator with domain and range in X.