PACIFIC JOURNAL OF MATHEMATICS
Vol 114, No 2, 1984

CHARACTERIZATION OF HOMOGENEOUS SPACES AND THEIR NORMS

J. A. WARD

Let G denote a compact group and B a homogeneous Banach space of pseudomeasures over G (B is left translation invariant with continuous shifts). If $T(G)$ is the linear space of trigonometric polynomials defined on G then $T(G) \cap B$ is a dense subspace of B. An explicit description is given of $T(G) \cap B$. A complete list of the homogeneous closed subspaces of B is given. Moreover it is shown that B is determined by the set $T(G) \cap B$ and N, the restriction of the B norm to this set. This leads to a complete description of those subsets of $T(G)$ and norms N which determine homogeneous Banach spaces.

Introduction. Homogeneous Banach spaces first appeared in the paper of G. Silov, [**9**], where they were introduced as classes of Banach algebras of functions (under pointwise multiplication) on compact abelian groups. Each has the underlying group as its maximal ideal space. In [**6**], K. de Leeuw omitted this property before classifying all possible maximal ideal spaces of homogeneous Banach algebras, and determining a correspondence between these algebras and certain norms.

If pointwise multiplication becomes convolution (and the underlying group is required to be locally compact and abelian) then we have the Segal algebras introduced in [7] by H. Reiter. The generalization of Segal algebras to homogeneous Banach algebras given by H. C. Wang in [**10**] corresponds to de Leeuw's extension of Silov's algebras. J. T. Burnham generalized, in [**1**], Segal algebras to allow the underlying group to be non-abelian. See Burnham's discussion in [**2**] of the development of this subject.

In this paper we study homogeneous Banach spaces in which the underlying group is taken to be compact and non-abelian. The translation operator is taken to be the left translation operator. In §2 we establish some fundamental properties of these spaces; each is an $M(G)$-Banach module, 2.2, and further each has a set of trigonometric polynomials as a dense subspace, 2.4. In 2.6 we give an explicit description of this subspace.

We use these results in §3 to classify all the homogeneous subspaces of a homogeneous Banach space, 3.2. We also show in 3.3 that if every element of the space is a measure, then each element is in fact generated by an L^1-function. This allows us to conclude in 3.4 that every such space is actually a convolution algebra.