PACIFIC JOURNAL OF MATHEMATICS
Vol. 114, No 2, 1984

TOPOLOGICAL METHODS FOR C^*-ALGEBRAS IV: MOD P HOMOLOGY

CLAUDE SCHOCHET

Let h_* be a homology theory on an admissible category of C^*-algebras. We define a homology theory $h_*(-;\mathbf{Z}/n)$ which fits into a Bockstein exact sequence

$$\to h_j(A) \xrightarrow{n} h_j(A) \xrightarrow{\rho_n} h_j(A;\mathbf{Z}/n) \xrightarrow{\beta_n} h_{j-1}(A) \to \cdots .$$

Let p be a prime. If p is odd or if h_* is "good" then $h_*(A;\mathbf{Z}/p)$ is a $\mathbf{Z}/p$-module and (with finiteness assumptions on the torsion of $h_*(A)$) there is a Bockstein spectral sequence with $E^1_* = h_*(A;\mathbf{Z}/p)$ which converges to $(h_*(A)/(\text{torsion})) \otimes \mathbf{Z}/p$. In the special case of K-theory, we show that $K_*(A \otimes N) \cong K_*(A;\mathbf{Z}/n)$, provided that $K_0(N) = \mathbf{Z}/n$, $K_1(N) = 0$, and N is in a certain (large) category $\mathfrak{N}$ of separable nuclear C^*-algebras.

Let h_* be a homology theory on an admissible category of C^*-algebras. This paper has three objects: to define and investigate the properties of the associated mod p homology theory $h_*(-;\mathbf{Z}/p)$, to generalize the notion of Bockstein coboundary homomorphisms and the apparatus of the Bockstein spectral sequence to this setting, and to establish a uniqueness theorem for the introduction of mod p coefficients into K-theory.

DEFINITION [16]. A *homology theory* is a sequence $\{h_n\}$ of covariant functors from an admissible category $\mathcal{C}$ of C^*-algebras to abelian groups which satisfies the following axioms:

Homotopy axiom. Let $h\colon A \to C([0,1], B)$ be a homotopy from $f_0 = p_0 h$ to $f_1 = p_1 h$ $[p_i(\xi) = \xi(i)]$ in $\mathcal{C}$. Then $f_{0*} = f_{1*}\colon h_n(A) \to h_n(B)$ for all n.

Exactness axiom. Let

$$0 \to J \xrightarrow{i} A \xrightarrow{j} B \to 0$$

be a short exact sequence in $\mathcal{C}$. Then there is a map $\partial\colon h_n(B) \to h_{n-1}(J)$ and a long exact sequence

$$\to h_n(J) \xrightarrow{i_*} h_n(A) \xrightarrow{j_*} h_n(B) \xrightarrow{\partial} h_{n-1}(A) \to \cdots .$$