PACIFIC JOURNAL OF MATHEMATICS
Vol. 114, No. 2, 1984

PROJECTIVE SPACE AS A BRANCHED COVERING OF THE SPHERE WITH ORIENTABLE BRANCH SET

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1. Introduction. Let M be a closed, orientable PL n-manifold. A theorem of Alexander [2] states that every such manifold is a piecewise linear branched covering of the n-sphere, S^n, i.e. there is a finite-to-one open PL map $f\colon M \to S^n$. The subset of M where f fails to be a local homeomorphism is called the singular set and the image of the singular set is called the branch set. Brand [4] suggests the problem of determining the values of n for which $\mathbf{R}P^n$, real projective n-space, is a branched covering of S^n with branch set a locally flat submanifold of S^n, and he shows that if such a covering exists, then $n = 2^t \pm 1$. We show that the values of n can be further limited if the branch set is orientable.

THEOREM 1.1. *If $\mathbf{R}P^n$ is a branched covering of S^n with locally flat, orientable branch set, then $n = 1, 3$, or 7.*

The converse of Theorem 1.1 is true in the cases $n = 1$ or 3: the identity map provides a branched covering of S^1 and Hilden and Montesinos have shown, independently, that every closed, orientable 3-manifold is a branched covering of S^3 with branch set a locally flat 1-manifold, [6] and [9]. Theorem 1.1 shows that if the branch set is required to be orientable, $n = 7$ is the only open case.

2. Normalized branched coverings. In [5], Brand proves a normalization theorem for smooth branched coverings. He uses his normalization theorem to show that there is a certain K-theoretic necessary condition for the existence of smooth branched coverings. In [4], Brand extended his normalization theorem to branched coverings with locally flat branch sets. He then showed that a branched covering with locally flat branch set is the pull-back of a universal smooth branched covering and hence must satisfy the same K-theoretic necessary conditions as a smooth branched covering.

If η is a 2-plane bundle over a complex X, let $\mu_k(\eta)$ be the 2-plane bundle obtained from η by the homomorphism $\mu_k\colon \mathrm{O}(2) \to \mathrm{O}(2)$ given by