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CHARACTERS OF INDUCED REPRESENTATIONS AND WEIGHTED ORBITAL INTEGRALS

REBECCA A. HERB

The main result of this paper is a formula relating characters of principal series representations of a reductive Lie group to weighted orbital integrals of wave packets.

1. Introduction. Let G be a reductive Lie group satisfying Harish-Chandra's general assumptions [**2**]. Let $P = MAN$ be the Langlands decomposition of a cuspidal parabolic subgroup of G. Denote by $\varepsilon_2(M)$ the set of equivalence classes of irreducible unitary square integrable representations of M. For $\omega \in \varepsilon_2(M)$ and $\nu \in \mathscr{F} = \mathfrak{a}^*$, the real dual of the Lie algebra of A, let $\pi_{\omega,\nu}$ be the corresponding unitary representation of G induced from P. Let f be a wave packet corresponding to ω. Then the integral of f over any regular (semisimple) orbit of G which can be represented by an element of $L = MA$ has been evaluated by Harish-Chandra in terms of the character $\Theta_{\omega,\nu}$ of $\pi_{\omega,\nu}$ [**4**].

Let γ be a regular element of G contained in a Cartan subgroup H of L. Write $H = H_K H_p$ where H_K is compact, H_p is split, and $A \subseteq H_p$. Then for suitable normalizations of the G-invariant measure $d\dot{x}$ on $H_p \setminus G$ and Haar measure $d\nu$ on $\mathscr{F}$.

$$(1.1)\quad \int_{H_p\setminus G} f(x^{-1}\gamma x)\, d\dot{x} = \varepsilon(A, H)[W(\omega)]^{-1}\int_{\mathscr{F}} \langle \Theta_{\omega,\nu}, f\rangle \Theta_{\omega,\nu}(\gamma)\, d\nu$$

where $W(\omega) = \{s \in N_G(A)/L \mid s\omega = \omega\}$ and $\varepsilon(A, H)$ is 1 if $H_p = A$ and is 0 otherwise. This formula can be interpreted as giving the value of $\Theta_{\omega,\nu}$ on regular elements γ of a fundamental Cartan subgroup of L in terms of the integral of a wave packet for ω over the orbit of γ. It also gives the Fourier inversion formula for the tempered invariant distribution

$$f \to \langle \Lambda(\gamma), f\rangle = \int_{H_p\setminus G} f(x^{-1}\gamma x)\, d\dot{x}$$

restricted to the subspace of $\mathscr{C}(G)$, the Schwartz space of G, spanned by wave packets corresponding to representations induced from cuspidal parabolic subgroups $P = MAN$ with $A \subseteq H_p$. The complete Fourier inversion formula for $\Lambda(\gamma)$ is much more complicated. (See [**5**].)