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CONVERGENCE OF ADAPTED SEQUENCES OF PETTIS-INTEGRABLE FUNCTIONS

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When considering adapted sequences of Pettis-integrable functions with values in a Banach space we are dealing with the following problem: when do we have a strongly measurable Pettis-integrable limit? Here the limit can be taken in the strong or weak sense a.e. or in the sense of the Pettis-topology.

Not many results in this area are known so far.

In this paper we give some pointwise convergence results of martingales, amarts, weak sequential amarts and pramarts consisting of strongly measurable Pettis-integrable functions. Also the Pettis convergence result of Musiał for amarts is extended.

The results are preceded by a preliminary study of some vector measure notions such as Pettis uniform integrability and σ-bounded variation. We give a new proof of the result of Thomas stating that in every infinite dimensional Banach space one can find a vector measure which is not of σ-bounded variation.

1. Introduction, terminology and notation. In the sequel, E will be a Banach space and $(\Omega, \mathscr{F}, P)$ a fixed complete probability space. A function $X: \Omega \to E$ is called scalarly measurable if $\langle x', X\rangle$ is measurable for each $x' \in E'$.

A function X is called Pettis-integrable if it is scalarly integrable and if for each $A \in F$, there exists $x_A \in E$ such that, for each $x' \in E'$

$$\langle x', x_A\rangle = \int_A \langle x', X\rangle dP.$$

x_A is denoted by $\int_A X\,dP$, the Pettis-integral of X over A. Let X and Y be two Pettis-integrable functions. We say that X is weakly equivalent with Y, denoted by $X \sim Y$, if for each $x' \in E'$

$$\langle x', X\rangle = \langle x', Y\rangle, \text{ a.e.}$$

Denote by P_E the space of all Pettis-integrable functions up to weak equivalence. Put on P_E the following norm, called Pettis-norm,

$$\|\cdot\|_P: P_E \to R^+, \quad X \to \|X\|_P = \sup_{\substack{\|x'\| \le 1 \\ x' \in E'}} \int_\Omega |\langle x', X\rangle| dP.$$