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* PRODUCTS AND REPRESENTATIONS OF NILPOTENT GROUPS

D. ARNAL

On each orbit W of the coadjoint representation of a nilpotent, connected and simply connected Lie group G, there exist * products which are relative quantizations for the Lie algebra $\mathfrak{g}$ of G. Choosing one of these * products, we first define a *-exponential for each X in $\mathfrak{g}$. These *-exponentials are formal power series and, with the * product, they form a group. Thanks to that, we are able to define a representation of G in a "* polarization" and to intertwine it with the unitary irreducible one associated to W. Finally, we study the uniqueness of our construction.

1. Introduction. The mathematical signification of quantization was specified by Bayen, Flato, Fronsdal, Lichnerowicz and Sternheimer with the theory of deformations of the associative algebra of C^∞ functions on the symplectic manifold W defined by the classical system [**3**]. (The principal results of the theory will be given in §2 for completeness.)

Previously, some other methods of geometrical quantization were considered by Kiriilov [**7, 9, 16, 8**]. This last approach had a very important link with questions of finding and classifying unitary irreducible representations of a group G. The easiest and the most complete case is of course when G is nilpotent. Let us suppose G is nilpotent, connected and simply connected. We know all its unitary irreducible representations [**8, 15**]. There exists a one-to-one mapping between classes of these representations U and orbits W of the coadjoint representation of G. On the other hand, the geometrical quantization of W, i.e. the construction of fibre bundles with base W and fibre a circle, is unique, the de Rham cohomology of W being trivial. Moreover, the representation U can be canonically defined with that quantization and a polarization [**7**].

It is tempting to "test" the method of quantization by deformation (* quantization) on the problem of constructing and classifying unitary irreducible representations of connected, simply connected, nilpotent groups.

The goal of this work is to canonically find the unitary irreducible representation associated to an arbitrary orbit W by means of * products defined on W. We first recall the principal definitions and results of the theory of * products (which are formal deformations with parameter λ of