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MULTIPLE SERIES ROGERS-RAMANUJAN TYPE IDENTITIES

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It is shown how each of the classical identities of Rogers-Ramanujan type can be embedded in an infinite family of multiple series identities. The method of construction is applied to four of L. J. Rogers' elegant series related to the quintuple product identity. Other applications are also presented.

1. Introduction. The Rogers-Ramanujan identities [**6**; Ch. 7] are given analytically as follows:

$$(1.1)\qquad 1+\sum_{n=1}^{\infty}\frac{q^{n^2}}{(1-q)(1-q^2)\cdots(1-q^n)}=\prod_{n=0}^{\infty}\frac{1}{(1-q^{5n+1})(1-q^{5n+4})},$$

$$(1.2)\qquad 1+\sum_{n=1}^{\infty}\frac{q^{n^2+n}}{(1-q)(1-q^2)\cdots(1-q^n)}=\prod_{n=0}^{\infty}\frac{1}{(1-q^{5n+2})(1-q^{5n+3})}.$$

Numerous authors [**18**], [**19**], [**16**], [**12**], [**13**], [**22**], [**23**] in the first half of this century found related results connecting q-series resembling those in (1.1) and (1.2) with various modular forms and functions. The culmination of their efforts may be found in the two papers of L. J. Slater [**22**], [**23**] wherein over 130 such identities are cataloged.

Within the last decade it has been observed that if one extends the q-series allowed to multiple series then infinite families of Rogers-Ramanujan type identities can be found [**4**], [**5**], [**14**], [**17**], [**25**], [**26**]. For example [**4**],

$$(1.3)\qquad \sum_{n_{k-1}\geq\cdots\geq n_1\geq 0}\frac{q^{n_1^2+n_2^2+\cdots+n_{k-1}^2}}{(q)_{n_{k-1}-n_{k-2}}(q)_{n_{k-2}-n_{k-3}}\cdots(q)_{n_1}}=\prod_{\substack{n=1\\ n\not\equiv 0,\,\pm k\,(\mathrm{mod}\,2k+1)}}^{\infty}(1-q^n)^{-1},$$