BIMEASURE ALGEBRAS ON LCA GROUPS

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For locally compact abelian groups G_1 and G_2, with character groups Γ_1 and Γ_2, respectively, let $BM(G_1, G_2)$ denote the Banach space of bounded bilinear forms on $C_0(G_1) \times C_0(G_2)$. Using a consequence of the fundamental inequality of A. Grothendieck, a multiplication and an adjoint operation are introduced on $BM(G_1, G_2)$ which generalize the convolution structure of $M(G \times H)$ and which make $BM(G_1, G_2)$ into a K_G^2-Banach *-algebra, where K_G is Grothendieck's universal constant. The Fourier transforms of elements of $BM(G_1, G_2)$ are defined and characterized in terms of certain unitary representations of Γ_1 and Γ_2. Various aspects of the harmonic analysis of the algebras $BM(G_1, G_2)$ are studied.

Introduction. Let S be the space of all doubly-indexed, bilateral, complex sequences of the form $\langle U_1^m \xi, U_2^n \eta \rangle$, where U_1 and U_2 are unitary operators on a Hilbert space H and $\xi, \eta \in H$. In [17] it was shown that, under coordinatewise addition and multiplication, S is an algebra containing all sequences of Fourier-Stieltjes coefficients of complex Borel measures on the torus T^2. It was also show that if

$$V = C(T) \mathbin{\hat{\otimes}} C(T)$$

denotes the projective tensor product of the space $C(T)$ with itself, then there is a natural embedding of S in the dual V^* of V. Namely, if $(\alpha_{mn}) \in S$ there is a unique element $u \in V^*$ such that

$$\alpha_{mn} = \left\langle e^{-in\theta} \otimes e^{-im\phi}, u \right\rangle, \qquad -\infty < m, n < \infty.$$

The question whether every element of V^* arises from S in this way was left open in [17]. However, it was pointed out to us by G. Pisier that a positive answer to this question follows easily from the Fundamental Theorem of the Metric Theory of Tensor Products (Theorem 1.2 below) of A. Grothendieck. (See Theorem 2.4(i).) It is a pleasure to express our gratitude to him here for having communicated this fact to us and to T. Ito for a number of helpful conversations with the second author. The purpose of this paper is to extend these ideas to the context of all locally compact abelian (LCA) groups and to examine some of their ramifications. That is, we wish to initiate the study of the harmonic analysis of the space $[C_0(G_1) \mathbin{\hat{\otimes}} C_0(G_2)]^*$ of bimeasures on a pair G_1, G_2 of LCA groups.