PACIFIC JOURNAL OF MATHEMATICS
Vol. 115, No. 2, 1984

ON SEMIGROUPS OF CONVOLUTION OPERATORS IN HILBERT SPACE

JORGE D. SAMUR

Given an infinitely divisible probability measure on a real separable Hilbert space H and the infinitesimal generator A of the associated semigroup of convolution operators acting on the Banach space of bounded uniformly continuous real functions on H, we describe the action of A on certain classes of differentiable functions.

1. Introduction. For every infinitely divisible probability measure μ on a real separable Banach space E there is an associated strongly continuous semigroup of convolution operators on the Banach space $C_u(E)$, the class of bounded uniformly continuous real-valued functions on E with the norm of uniform convergence. According to the general theory of semigroups of operators, the domain of the infinitesimal generator of every such semigroup is dense in $C_u(E)$. As is well known, one of the central aspects of the study of a specific semigroup of operators is the description of the action of its infinitesimal generator on a class of "smooth" functions which is large enough to characterize the semigroup. In the case when E is finite-dimensional, a result of this kind was obtained by Courrège [3], where the action of all generators of convolution semigroups on a natural class of differentiable functions is described. When E is an infinite-dimensional Banach space, however, the scarcity of differentiable functions (see [*] for a recent discussion) does not allow such a description.

This difficulty can be surmounted in the case when E is a Hilbert space; this is the object of the present paper. We consider the case in which E is a Hilbert space H and describe the action of the generators on certain classes of differentiable functions. We exhibit a natural class of differentiable functions — the class $C_u^{(2)}(H)$, defined below — on which all generators of convolution semigroups can be characterized (Theorem 3.1); our result generalizes the work of Courrège [3]. However, in contrast to the situation in the finite-dimensional case, $C_u^{(2)}(H)$ is not dense in $C_u(H)$ when H is infinite-dimensional.

It is possible to prove a stronger result for convolution semigroups without Gaussian component; in fact, in Theorem 3.6 we describe the