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CARATHEODORY CONVEX INTEGRAND OPERATORS AND PROBABILITY THEORY

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In two recent papers, the author studied extensions of several concepts of nonsmooth analysis to vector valued operators. The purpose of the present work is to further continue this effort and to study, from a probabilistic viewpoint, several properties of convex operators. In particular, we will examine how various basic concepts of vectorial nonsmooth analysis associated with an integrand $f(\omega, x)$ are related to those of the integral operator $F(x) = \int_\Omega f(\omega, x)\, d\mu(\omega)$ where the vector valued integral is defined in the sense of Bochner. Also we introduce a conditional expectation for such integrands, study several of its properties, see how it is affected by various operations of nonsmooth analysis, and derive a vector valued martingale convergence theorem.

1. For real valued functions, the most important contributions on this subject were made by Rockafellar [**17, 18, 19, 20**], who introduced the notion of the normal integrand. Caratheodory integrands form a subset in the family of normal integrands. Following Rockafellar, interesting results were also obtained by Bismut [**3**] and Hiriart-Urruty [**10**]. Our work generalizes several of their results to a vector valued context.

In our presentation, we will use parts of the general theory of ordered topological vector spaces. For the necessary background on this topic, we refer the reader to the books by Peressini [**16**] and Schaefer [**23**].

2. Preliminaries. We start by recalling some basic facts from the theory of measurable multifunctions. For more details the reader can refer to Castaing-Valadier [**4**], Himmelberg [**9**] or Rockafellar [**20**].

Let $F\colon \Omega \to 2^X$ be a multivalued function (multifunction) from a space to the family of subsets of a space X. We introduce the set

$$\operatorname{Gr} F = \{(\omega, x) \in \Omega \times X\colon x \in F(\omega)\}$$

which we call the graph of F. Also for $V \subseteq X$, we define $F^{-}(V) = \{\omega \in \Omega\colon F(\omega) \cap V \neq \varnothing\}$. If X is a topological space, by $P_f(X)$ we denote the nonempty and closed subsets of X.

The next theorem summarizes the interrelations of the different notions of measurability of a multifunction that exist in the literature.