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UNCONDITIONAL BASES AND FIXED POINTS OF NONEXPANSIVE MAPPINGS

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We prove that every Banach space with a 1-unconditional basis has the fixed point property for nonexpansive mappings. In fact the argument works if the unconditional constant is $< (\sqrt{33} - 3)/2$.

1. Introduction. Let K be a weakly compact convex subset of a Banach space X. We say K has the *fixed point property* if every *nonexpansive* map $T: K \to K$ (i.e. $\|Tx - Ty\| \leq \|x - y\|$ for $x, y \in K$) has a fixed point. We say X has the fixed point property if every weakly compact convex subset of X has the fixed point property.

It is known that L_1 fails the fixed point property [**A**]. On the other hand, Kirk [**Ki 1**] proved that every Banach space with normal structure (for the definition see [**D**]) has the fixed point property. Karlovitz (see [**Ka 1**] and [**Ka 2**]) extended Kirk's work. Let us explain what Karlovitz did.

Suppose K is weakly compact convex and $T: K \to K$ is nonexpansive. K contains a weakly compact convex subset K_0 which is *minimal* for T. This means $T(K_0) \subseteq K_0$ and no strictly smaller weakly compact convex subset of K_0 is invariant under T. If K_0 contains only one point, then T has a fixed point. Hence, we may assume that $\operatorname{diam} K_0 = \sup\{\|x - y\|: x, y \in K_0\} > 0$. It is easy to see that K_0 contains a sequence (x_n) with $\lim_{n\to\infty}\|x_n - Tx_n\| = 0$. We call such a sequence an *approximate fixed point sequence* for T. Indeed, fixed $y \in K_0$, one can choose x_n to be the fixed point of the strict contraction, $T_n: K_0 \to K_0$, given by $T_n x = (1 - n^{-1})Tx + n^{-1}$. Note we only need that K_0 is closed, bounded and convex for this argument. Karlovitz proved the following theorem.

THEOREM A. *Let K be a minimal weakly compact convex set for a nonexpansive map T, and let (x_n) be an approximate fixed point sequence. Then for all $x \in K$*

$$\lim_{n\to\infty} \|x - x_n\| = \operatorname{diam} K.$$

Maurey [**M**] used the ultraproduct techniques to prove that c_0 and every reflexive subspace of L_1 have the fixed point property. Odell and the