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FINITE GROUP ACTION AND EQUIVARIANT BORDISM

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Conner and Floyd proved that if $\mathbf{Z}_2^k$ acts on a closed manifold M differentiably and without any fixed point, then M is a boundary. Stong gave a stronger result proving that if (M, θ) is a closed $\mathbf{Z}_2^k$-differential manifold with no stationary point, then (M, θ) is a $\mathbf{Z}_2^k$-boundary. In the present note, we discuss this problem for a finite group in detail. Let G be a finite group. By the 2-central component $G_2(C)$ of G, we will mean the subgroup of G consisting of the identity element and all the elements of order 2 in the center of G. We prove in this note that the fixed data of the 2-central component $G_2(C)$ of G determines G-bordism.

1. Preliminaries. Throughout the note we will take G to be a finite group. By a G-manifold we will mean a differential compact manifold with a differential action of G on it. A family $\mathscr{F}$ in G is a collection of subgroups of G such that if $H \in \mathscr{F}$, then all the subgroups of H and all the conjugates of H are in $\mathscr{F}$. Let $\mathscr{F}' \subset \mathscr{F}$ be families in G such that $\exists$ a central element a in G of order 2 such that

(i) $a \notin H, \forall H \in \mathscr{F} - \mathscr{F}'$

(ii) $H \in \mathscr{F}' \Rightarrow [H \cup \{a\}] \in \mathscr{F}'$

(iii) The intersection S of all members of $\mathscr{F} - \mathscr{F}'$ is in $\mathscr{F} - \mathscr{F}'$. We call such a pair $(\mathscr{F}, \mathscr{F}')$ of families an admissible pair of families in G with respect to $a \in G$.

EXAMPLE 2.1. Let G be a finite group. We can write the 2-central component $G_2(C)$ as $\mathbf{Z}_2^r = [t_1, \ldots, t_r]$, where $t_1, \ldots, t_r$ are generators of $\mathbf{Z}_2^r$ with $t_i^2 =$ the identity element and $t_i t_j = t_j t_i$. Let $\mathscr{F}_k$ be the family of all subgroups of G not containing $\mathbf{Z}_2^k$, $0 < k \leq r$, where $\mathbf{Z}_2^k$ denotes the subgroup of G generated by the first k generators $t_1, \ldots, t_k$. Then $(\mathscr{F}_{k+1}, \mathscr{F}_k)$ is an admissible pair with respect to t_{k+1}, $0 < k < r$.

2. Stationary point free action of $G_2(C)$ and G-bordism. The object of this section is to show that if (M, θ) is a G-manifold with the stationary point free action of $G_2(C)$ then (M, θ) is G-boundary. Following the notation of Stong [**2**], let $\mathfrak{N}_*(G; \mathscr{F}, \mathscr{F}')$ denote the $(\mathscr{F}, \mathscr{F}')$-free G-bordism group for a pair $(\mathscr{F}, \mathscr{F}')$ of families in G. For a given family $\mathscr{F}$