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DECOMPOSITIONS OF ALGEBRAICALLY COMPACT MODULES

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This paper is mainly concerned with describing the category of all algebraically compact (= pure-injective) modules. A family of functors from this category to categories of injective modules, that is, spectral categories, is defined. Via these functors we transfer the decompositions of the objects of a spectral category and their invariants to algebraically compact modules. For instance, as a corollary we find the decompositions and the invariants for algebraically compact abelian groups and the decompositions for algebraically compact modules over Prüfer rings. Our results yield a connection between the theory of algebraically compact modules and the one of injective modules.

The theory of algebraically compact abelian groups, mainly due to Kaplansky [**12**], Łoś [**13**], Balcerzyk [**2**] and Maranda [**14**], has been extended to modules by Stenström [**18**] and Warfield [**19**]. Recently the structure of algebraically compact modules has been studied with methods of model theory by Fisher [**6**], Garavaglia [**9**], [**10**], and Ziegler [**21**]. It was Maranda [**14**] who first pointed out the similarity between the theory of algebraically compact abelian groups and that of divisible (injective) groups. Later Warfield implicitly asked in the last paragraph of [**20**] whether it was possible to fit his methods and results on injective modules to algebraically compact ones.

In this paper we answer Warfield's question and study algebraically compact modules in the context of categories of injectives, i.e. spectral categories. Spectral categories (Grothendieck categories in which every exact sequence splits) were introduced by Gabriel and Oberst [**8**] and studied by Roos [**17**], Goodearl and Boyle [**11**], where a beautiful dimension theory for the objects of a spectral category was constructed, and the author [**5**].

After the preliminaries of the first section, we define a family of functors F_E from the full subcategory of Mod-R generated by all algebraically compact modules into suitable spectral categories. This is done in section two by using the powerful results on algebraically compact modules obtained by Zimmermann-Huisgen and Zimmermann in [**22**]. In section three we show that our functors F_E map pure-injective envelopes