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p-ADIC OSCILLATORY INTEGRALS AND WAVE FRONT SETS

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For K a p-adic field, we examine oscillatory integrals

$$I(\phi, p)(\lambda) = \int_{K^n} \phi(x)\Psi(\lambda p(x))\,dx$$

where ϕ is a Schwartz function on K^n, Ψ is an additive character, $\lambda \in K^x$, and $p\colon K^n \to K$ is locally analytic. If $Dp \neq 0$ on the support of ϕ, $\lambda \mapsto I(\phi, p)(\lambda)$ has bounded support. If $Dp(x_0) = 0$ at exactly one point x_0 in the support of ϕ but $D^2p(x_0)$ is non-degenerate, then

$$I(\phi, p)(\lambda) = |\lambda|^{-n/2}\gamma\Psi(p(x_0))|\det D^2p(x_0)|^{-1/2}\phi(x_0)$$

for sufficiently large $|\lambda|$, where γ is a complex eighth root of unity. An invariant definition of wave front set, $WF_\Lambda(u)$, for distributions u relative to an open subgroup Λ of $K^\times$ is proved to exist, analogous to the classical case, with "rapidly decreasing" replaced by "bounded support". Definitions of pull backs and push forwards of distributions, distribution products, and kernel maps are made, again similar to the classical case, and their wave front sets computed. Wave front sets $WF_\Lambda(\rho)$ of representations ρ of p-adic groups are also defined (cf. Howe, Automorphic forms, representation theory, and arithmetic, Tata Inst., 1979, for the Lie group analogue). For admissible representations ρ of, say, a semi-simple group G, with character χ_ρ, we show that $WF_\Lambda^0(\rho) = WF_\Lambda^0(\chi_\rho)$, where $WF_\Lambda^0(\cdot) \subseteq \mathrm{Lie}(G)$ is $WF_\Lambda(\cdot)$ above the identity element. Functorial properties of $WF_\Lambda(\rho)$ are developed and examples computed.

Introduction. The motivation for this work is to apply to p-adic groups the approach of Howe [H] who has applied classical wave front set theory to Lie group representations. Chapter I develops a stationary phase formula for p-adic oscillatory integrals. In Chapter II, p-adic wave front sets are defined, relative to multiplicative subgroups of the multiplicative group of a p-adic field. The wave front set of a representation of a p-adic group is then defined and developed in Chapter III.

We denote by K a locally compact field of characteristic 0 with valuation $|\cdot|_K$, O_K its ring of integers, and P_K the unique maximal prime ideal of O_K. Denote by ϖ a fixed uniformizing element. If $|\varpi|_K = q^{-1}$, define $\mathrm{Ord}(x)$, $x \in K^\times$, by $|x| = q^{-\mathrm{Ord}(x)}$. On the n-dimensional space K^n, we use the norm $|(x_1,\ldots,x_n)|_{K^n} = \max\{|x_i|_K\}$. The unit sphere is denoted Σ^{n-1}. Identify K^n with its dual $(K^n)^*$ by the symmetric bilinear form $\{x, y\} = \Sigma_{i=1}^n x_i y_i$. Fix an additive character Ψ of K with conductor O_K.