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THE ORDERING STRUCTURE ON BANACH SPACES

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Suppose X, Y are Banach spaces. The binary relation $X \prec Y$ means $X = \bigcap T^{**-1}[Y]$, where the intersection is taken over all bounded linear operators $T\colon X \to Y$. We use this definition to study sums of Banach spaces and the spaces $J(\omega_1)$, $C_{[0,\omega_1]}$.

Introduction. In this paper we use the binary relation defined by G. A. Edgar in [**5**] to investigate some relationships among some sequence Banach spaces.

Notation and terminology used in this paper follow [**5**] and match Diestel and Uhl [**1**], Dunford and Schwartz [**2**], Kelley [**8**], Lindenstrauss and Tzafriri [**9**]. If X is a Banach space, its dual will be denoted X^*, its bidual X^{**}. The subset of X^{**} canonically identified with X will simply be written X.

In [**5**], G. A. Edgar established a binary relation " $\prec$ " for Banach spaces which is defined by

DEFINITION. Let X and Y be Banach spaces. Then $X \prec Y$ means

$$X = \bigcap T^{**-1}[Y]$$

where the intersection is taken over all bounded linear operators T: $X \to Y$.

Let $F(X, Y) = \bigcap T^{**-1}[Y]$. $F(X, Y)$ is called the Frame.

[**5**] points out that the definition can be rephrased as follows: $X \prec Y$ if and only if any $\alpha \in X^{**}$, such that $T^{**}(\alpha) \in Y$ for all bounded linear operators T: $X \to Y$, must be in X.

In this paper we use this ordering to consider the c_0-sum of Banach spaces, l_1-sum of Banach spaces and to compare them with c_0 space, $l_1(\Gamma)$ space respectively. We find out that under some natural conditions the ordering is preserved (see Propositions 3,5. More generally also see Proposition 4.) In this paper we also consider some function Banach spaces. We find out that the long James space, $J(\omega_1)$ is a predecessor of the continuous function space $C_{[0,\omega_1]}$ in this ordering (see Proposition 7), but $J(\omega_1)$ and $C_{[0,\omega_1]}$ both are not predecessors of l_∞ (see Propositions 8,11).