PACIFIC JOURNAL OF MATHEMATICS
Vol 116, No 2, 1985

ISOMORPHISMS OF SPACES OF NORM-CONTINUOUS FUNCTIONS

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If X and Y are compact Hausdorff spaces and E a uniformly convex Banach space, then the existence of an isomorphism T of $C(X, E)$ onto $C(Y, E)$ with $\|T\|\,\|T^{-1}\|$ small implies that X and Y are homeomorphic.

1. Introduction. Throughout this article, the letters X, Y, Z, and W will denote compact Hausdorff spaces, and E a Banach space. $C(X, E)$ denotes the space of continuous functions on X to E provided with the supremum norm. If E is a dual space then $C(X, E_{\sigma^*})$ stands for the Banach space of continuous functions F on X to E when this latter space is provided with its weak* topology, again normed by $\|F\|_\infty = \sup_{x\in X}\|F(x)\|$. If E is the one-dimensional field of scalars then we write $C(X)$ for $C(X, E)$. The interaction between elements of a Banach space and those of its dual is denoted by $\langle\ \cdot\ ,\cdot\rangle$. We write $E_1 \cong E_2$ to indicate that the Banach spaces E_1 and E_2 are isometric.

The well known Banach-Stone theorem states that if $C(X)$ and $C(Y)$ are isometric then X and Y are homeomorphic. Various authors, beginning with M. Jerison [**13**], have considered the problem of determining geometric properties of E which allow generalizations of this theorem to spaces of norm-continuous vector functions $C(X, E)$. The most exhaustive compilation of results of this nature can be found in the monograph by E. Behrends [**2**]. Another type of generalization of the theorem was obtained independently in [**1**] and [**3**], and, while still dealing with scalar functions, replaces isometries by isomorphisms T with $\|T\|\,\|T^{-1}\|$ small.

The first attempt to combine these two directions of generalization is found in [**4**], where it is shown that if E is a finite-dimensional Hilbert space, then the existence of an isomorphism T of $C(X, E)$ onto $C(Y, E)$ with $\|T\|\,\|T^{-1}\| < \sqrt{2}$ implies that X and Y are homeomorphic. More recently, K. Jarosz [**12**] has obtained a similar generalization for Banach spaces E whose dual space satisfies a geometric condition involving both $\|T\|\,\|T^{-1}\|$ and the number $4/3$. Here we obtain such a theorem for all uniformly convex spaces E. Moreover, given such a space E, the bound on the isomorphisms for which our theorem works depends on the modulus of convexity associated with E.