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SOME MAXIMUM PROPERTIES FOR A FAMILY OF SINGULAR HYPERBOLIC OPERATORS

LU ZHU-JIA

We study some maximum properties of solutions of the equation

$$L_{p,q,c}u \equiv u_{xx} - h^2(x)u_{tt} + ph'(x)u_t + q\frac{h'(x)}{h(x)}u_x + c(x,t)u = 0$$

with real parameters p and q. Some of the results here improve those of L. E. Payne and D. Sather. We also point out that a certain condition given by S. Agmon, L. Nirenberg and M. H. Protter is not only sufficient in order to obtain a kind of maximum property, but also necessary for a special case of $L_{p,q,c}$.

1. Introduction. Since the maximum principles were first established for a class of linear second order hyperbolic operators in two independent variables [**1**], [**3**], many authors have studied various maximum and monotonicity properties of some problems for classes of linear second order hyperbolic operators in two or more independent variables [**5**]–[**10**]. Later Payne and Sather considered a singular hyperbolic operator [**4**]. They obtained some maximum, monotonicity and convexity properties, as well as pointwise bounds, for the solution of some Cauchy and initial-boundary value problems for the Chaplygin operator

$$L \equiv \frac{\partial^2}{\partial x^2} - h^2(x)\frac{\partial^2}{\partial t^2}, \tag{1.1}$$

where h satisfies

$$\text{(a) } h \in C^1(R_+) \cap C^0(\overline{R}_+), \quad \text{(b) } h(0) = 0, \quad \text{(c) } h'(x) > 0, x > 0. \tag{1.2}$$

For example, Theorem 1 in [**4**] states that if h satisfies (1.2) and

$$\lim_{x\to 0}\frac{h(x)}{h'(x)} = 0, \tag{1.3}$$

and if u satisfies the conditions

$$\text{(a) } u \in C^2(E \cup AB) \cap C^1(\overline{E}), \quad \text{(b) } u_x \le 0 \quad \text{on } AB, \quad \text{(c) } Lu \le 0 \quad \text{in } E, \tag{1.4}$$

then

$$u \le \max_{AB} u \quad \text{in } E, \tag{1.5}$$