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A MULTILINEAR GENERATING FUNCTION FOR THE KONHAUSER SETS OF BIORTHOGONAL POLYNOMIALS SUGGESTED BY THE LAGUERRE POLYNOMIALS

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The polynomial sets $\{Y_n^\alpha(x;k)\}$ and $\{Z_n^\alpha(x;k)\}$, discussed by Joseph D. E. Konhauser, are biorthogonal over the interval $(0,\infty)$ with respect to the weight function $x^\alpha e^{-x}$, where $\alpha > -1$ and k is a positive integer. The object of the present note is to develop a fairly elementary method of proving a general multilinear generating function which, upon suitable specializations, yields a number of interesting results including, for example, a multivariable hypergeometric generating function for the multiple sum:

$$(*)\qquad \sum_{n_1,\ldots,n_r=0}^{\infty} (m+n_1+\cdots+n_r)!\,Y^\alpha_{m+n_1+\cdots+n_r}(x;k) \cdot \prod_{i=1}^{r}\left\{\frac{Z_{n_i}^{\beta_i}(y_i;s_i)u_i^{n_i}}{(1+\beta_i)_{s_in_i}}\right\},$$

involving the Konhauser biorthogonal polynomials; here, by definition,

$$\alpha > -1;\quad \beta_i > -1;\quad k, s_i = 1,2,3,\ldots;\qquad \forall\, i \in \{1,\ldots,r\}.$$

1. Introduction. Joseph D. E. Konhauser ([5]; see also [4]) introduced two interesting classes of polynomials: $Y_n^\alpha(x;k)$ a polynomial in x, and $Z_n^\alpha(x;k)$ a polynomial in x^k, $\alpha > -1$ and $k = 1,2,3,\ldots$. For $k=1$, these polynomials reduce to the classical Laguerre polynomials $L_n^{(\alpha)}(x)$, and for $k = 2$ they were encountered earlier by Spencer and Fano [8] in the study of the penetration of gamma rays through matter and were discussed subsequently by Preiser [7]. Also [5, p. 303]

$$(1)\quad \int_0^\infty x^\alpha e^{-x} Y_m^\alpha(x;k) Z_n^\alpha(x;k)\,dx = \frac{\Gamma(kn+\alpha+1)}{n!}\delta_{mn},\qquad \forall\, m,n \in \{0,1,2,\ldots\},$$