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THE EXTENSION OF EQUI-UNIFORMLY CONTINUOUS FAMILIES OF MAPPINGS

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In previous papers, we discussed the extension of uniformly continuous real-valued mappings from subspaces of metric spaces and the same question for mappings into certain Banach spaces, such as $c_0(I)$ and $l_\infty(I)$. Since the extension of uniformly continuous mappings into $l_\infty(I)$ is equivalent to the extension of equi-uniformly continuous point bounded families of real-valued mappings, it is natural to ask about the extension of equi-uniformly continuous families which are not necessarily point-bounded. The present paper investigates this extension property and several related questions concerning the extension of uniformly continuous mappings with values in uniformly discrete spaces.

I. Definitions and notation. Assume that X and Y are uniform spaces. Then $U(X, Y)$ denotes the family of uniformly continuous mappings from X to Y. If Y is the real line R, then $U(X, Y)$ will simply be denoted by $U(X)$. Assume that S is a subset of X. Then the pair (S, X) has the *Y-extension property* if every member of $U(S, Y)$ can be extended to a member of $U(X, Y)$. If Y is the real line, we say that S is *U-embedded* in X. It is a well-known theorem of Katětov that every bounded member of $U(S)$ can be extended to a member of $U(X)$.

If D is a set, $l_\infty(D)$ is the set of all bounded real-valued functions on D with the supremum metric. If the pair (S, X) has the $l_\infty(D)$-extension property for every set D, we say that (S, X) has the *l_∞-extension property*. A straightforward translation shows that (S, X) has the l_∞-extension property if and only if every point-bounded equi-uniformly continuous subfamily of $U(S)$ can be extended to a point-bounded equi-uniformly continuous subfamily of $U(X)$. If every equi-uniformly continuous subfamily of $U(S)$ can be extended to an equi-uniformly continuous subfamily of $U(X)$, we say that S is *strongly U-embedded* in X.

If D is a set, $F(D, R)$ denotes the family of all real-valued functions on D with the metric defined by $d(f, g) = \sup\{|f(x) - g(x)| \wedge 1 : x \in D\}$. In addition, we define $\|f - g\| = \sup\{|f(x) - g(x)|: x \in D\}$, allowing the possible value $+\infty$. Using the definition of the metric space $F(D, R)$, one can show (i) that $l_\infty(D)$ is a uniform subspace of $F(D, R)$, and (ii) that S is strongly U-embedded in X if and only if the pair (S, X) has the $F(D, R)$-extension property for every set D.