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A PROBLEM ON CONTINUOUS AND PERIODIC FUNCTIONS

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Let $f(x)$ be continuous and of period one on the real line. If d_j, $j = 1, 2, \ldots, n$, are n numbers such that each $d_j - d_1$ is rational, then there are two rational numbers r and r' for which

$$f(r) \le f(r + d_j) \quad \text{and} \quad f(r') \ge f(r' + d_j), \qquad j = 1, 2, \ldots, n.$$

This problem was communicated to the author by K. L. Chung and P. Erdös.

1. Introduction. Let $f(x)$ be a real valued function. We say that $f(x)$ is of period one if

$$f(x + 1) = f(x) \quad \text{for } -\infty < x < \infty.$$

A problem (communicated by Chung and Erdös) asks that if $f(x)$ is continuous and of period one, and if $d_j, j = 1, 2, \ldots, n$, are n numbers, can one find a rational number r such that

$$f(r) \le f(r + d_j), \qquad j = 1, 2, \ldots, n.$$

In this note, we present the following partial solution.

THEOREM 1. *Let $f(x)$ be continuous and of period one. If d_j, $j = 1, 2, \ldots, n$, are n numbers such that each $d_j - d_1$ is rational, then there are two rational numbers r and r' for which*

$$(1) \quad f(r) \le f(r + d_j) \quad \text{and} \quad f(r') \ge f(r' + d_j), \qquad j = 1, 2, \ldots, n.$$

2. Uniform distribution. Let x be a positive number and let $[x]$ be the largest integer less or equal to x. By a theorem of Hardy and Wright [**1**, Theorem 445], we know that if θ is irrational then the points $(n\theta) = n\theta - [n\theta]$ are uniformly distributed in $(0, 1)$. In particular, the points $(n\theta)$ are dense in $(0, 1)$. Based on this theorem, we shall prove the following result.