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CAPILLARY SURFACES OVER OBSTACLES

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We consider the usual capillarity problem with the additional requirement that the capillary surface lies above some obstacle. This involves a variational inequality instead of a boundary value problem. We prove existence of a solution to the variational inequality and study the boundary regularity. In particular, global $C^{1,1}$-regularity is shown for a wider class of variational inequalities with conormal boundary condition.

Let $\Omega \subset \mathbf{R}^n$, $n \geq 2$, be a bounded domain with smooth boundary $\partial\Omega$ and let

$$(0.1) \qquad A = -D_i(a^i(p)),^1 \qquad a^i(p) = p_i \cdot \left(1 + |p|^2\right)^{-1/2}$$

be the minimal surface operator. Then we study the variational inequality

$$(0.2) \qquad \langle Au + H(x,u), v - u \rangle \geq 0 \quad \forall v \in K,$$
$$K := \{ v \in H^{1,\infty} | v \geq \psi \}$$

where

$$(0.3) \qquad \langle Au, \eta \rangle = \int_\Omega a^i(Du) \cdot D_i\eta \, dx + \int_{\partial\Omega} \beta\eta \, dH_{n-1}.$$

Here H describes a gravitational field, ψ is the obstacle and β is the cosine of the contact angle at the boundary. We make the assumption that

$$(0.4) \qquad H = H(x,t) \in C^{0,1}(\mathbf{R}^n \times \mathbf{R}), \qquad \beta \in C^{0,1}(\partial\Omega)$$

satisfy the conditions

$$(0.5) \qquad \frac{\partial H}{\partial t} \geq \kappa > 0$$

and

$$(0.6) \qquad |\beta| \leq 1 - a, \qquad a > 0.$$

Under these assumptions Gerhardt **[2]** showed, that (0.2) admits a solution $u \in H^{2,p}(\Omega)$, if we impose on ψ the further condition

$$(0.7) \qquad -a^i(D\psi) \cdot \gamma_i \geq \beta \quad \text{on } \partial\Omega$$

[1]Here and in the following we sum over repeated indices.