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GENERALIZED QUOTIENT MAPS THAT ARE INDUCTIVELY INDEX-σ-DISCRETE

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E. Michael recently showed that a continuous quotient s-map between metrizable spaces can be contracted onto a G_δ-set so that the resulting map is index-σ-discrete; i.e., one that preserves σ-discretely decomposable families. Because of the potential utility of this result in descriptive set theory, we give a refinement that is less dependent upon the behavior of open sets under the map. Several types of generalized quotient maps are defined, not necessarily continuous, and we show that these are precisely the maps that are "inductively" index-σ-discrete under certain conditions similar to the above. The inter-relationships among these maps are also described. We further show that when the given map has a nice property (such as Borel measurability), then the restriction can be defined on a similarly nice subset of the domain. An application is made to maps that preserve analytic metric spaces; and additional applications to the existence of Borel measurable inverses will be given elsewhere.

1. Introduction and statements of principal results. The reader is referred to the paper of E. Michael [**8**] for background, general properties and detailed definitions of base-σ-discrete and index-σ-discrete maps. However, for convenience, we will briefly summarize the definitions here. Throughout, X and Y denote arbitrary topological spaces, and maps f: $X \to Y$ are *not* assumed to be continuous unless stated so. The power set of X is denoted $\mathscr{P}(X)$, and $\mathbf{N}$ denotes the set of natural numbers.

Let $\mathscr{B} \subset \mathscr{P}(X)$. We say that $\mathscr{B}$ is a *base* for a collection of sets $\mathscr{A}$, if each member of $\mathscr{A}$ is the union of a subcollection of $\mathscr{B}$. Recall that a base (not necessarily open) for the open sets of X is called a *network* for X. The collection $\mathscr{B}$ is *discrete* in X, if each point of X has a neighborhood meeting at most one member of $\mathscr{B}$; it is *σ-discrete*, if $\mathscr{B} = \bigcup_{n \in \mathbf{N}} \mathscr{B}_n$ and each $\mathscr{B}_n$ is discrete. A map f: $X \to Y$ is *base-σ-discrete* if the image of each discrete collection in X has a σ-discrete base in Y (see [**8**, §2] and [**2**, §3]). If $\mathscr{B} = \langle B_\lambda \rangle_{\lambda \in \Lambda}$, then $\mathscr{B}$ is *index-discrete* in X if each $x \in X$ has a neighborhood meeting B_λ for at most one λ (equivalently, $\mathscr{B}$ is discrete and $B_\lambda \cap B_{\lambda'} = \varnothing$ whenever $\lambda \neq \lambda'$; $\langle B_\lambda \rangle_{\lambda \in \Lambda}$ is *σ-discretely decomposable* if there are index-discrete families $\langle B_\lambda^n \rangle_\lambda$, for each $n \in \mathbf{N}$, such that