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COMPACTOID AND COMPACT FILTERS

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We study compactoid and compact filters which generalize the concepts of convergent filters and compact sets. In particular, we investigate their properties in subregular and regular spaces, their localizations, and their countable variants. Several classical results follow (e.g., theorems of Tychonoff, Kuratowski, Choquet). More recent results on preservation of compactness (e.g., Smithson) and local compactness (e.g. Lambrinos) are extended and refined.

0. Introduction. Several topological properties of relations may be expressed in terms of some filters on the image space. Therefore, certain investigations of relations may be reduced to the study of filters, thus simplifying the arguments. There arise compactoid, compact, semiconvergent, locally compact, cocompact, subregular, regular and other filters. Compactoid and compact filters generalize both convergent filters and compact sets. Frequently they may be used where convergence or compactness of sets is too strong an assumption (see, for instance, the classical Theorem 7.10 of Kuratowski [**15**]). Compactoid filters find numerous applications in optimization, generalized differentiation, differential equations, fixed point theory (see e.g. [**20**] and [**2**] where compactoid filters are hidden under the guise of measure of noncompactness) and elsewhere, primarily in the context of existence results.

To our knowledge, the concept of compactoid filter appeared for the first time in [**24**] by Topsøe, formulated with the aid of nets in topological spaces.[1] Unfamiliar with this, two of us reintroduced compactoid filters in [**9**]. The same idea occurred independently to Penot; in [**20**] he presents an extensive collection of applications. Going back to the twenties one finds compact sequences of Urysohn [**25**]. They constitute a sequential counterpart of compactoid filters.

Compactoid and compact filters are also of considerable theoretical interest. They provide a broader comprehension of the notion of compactness and enable one to establish new and subtler results (for instance, on preservation of compactness).

[1]The notion of compactoid filter appears also in M. P. Kac, *Characterization of some classes of pseudotopological linear spaces*, in Convergence Structures and Applications to Analysis, Akademie-Verlag, Berlin, 1980, 115–135.