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AN APPROXIMATION THEOREM FOR EQUIVARIANT LOOP SPACES IN THE COMPACT LIE CASE

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Let V be a real orthogonal countable-dimensional representation of the Lie group G and denote by $\Omega^V\Sigma^V X$ the space of maps $S^V \to \Sigma^V X = X \wedge S^V$, where S^V denotes the one-point compactification of V and where X is an arbitrary G-space with stationary basepoint (if V is infinite-dimensional, $\Omega^V\Sigma^V X$ is taken as the natural colimit over spaces indexed on the finite-dimensional submodules of V). Since G acts on $\Omega^V\Sigma^V X$ by conjugation, the fixed-set $(\Omega^V\Sigma^V X)^G$ is the subspace of G-equivariant maps. We present here an approximation to $(\Omega^V\Sigma^V X)^G$ in the stable case (V large). This approximation will take the form of a space of "configurations" of G-orbits in V.

In the Geometry of Iterated Loop Spaces **[M1]**, J. P. May carries out this program using an approximation

$$\alpha_n \colon C_n X \to \Omega^n\Sigma^n X$$

for $1 \leq n$. The map α_n is a homotopy-equivalence when X is connected, and in general is a group-completion. This means that α_n is an H-map between an H-space and a group-like H-space, and $(\alpha_n)_*\colon H_*(C_n X) \to H_*(\Omega^n\Sigma^n X)$ is a localization of the ring $H_*(C_n X)$ at its multiplicative submonoid $\pi_0 X$ for field coefficients. (See **[M2**, Ch. 15].)

We wish to carry out such a program in the equivariant case, where all spaces are acted upon by a group G. The right space to approximate in place of $\Omega^n\Sigma^n X$ is the space $\Omega^V\Sigma^V X$. Such an approximation exists in the case where G is finite (**[H1]**, **[S2]**). But the case where G is a compact Lie group is much deeper, and our approximation to $(\Omega^V\Sigma^V X)^G$ is therefore not the greatest generality one could hope for. Indeed, a suitable approximation to $\Omega^V\Sigma^V X$ (even in the stable case) would suffice to prove an equivariant "regcognition principle"—a simple test enabling one to determine whether a given space admits deloopings of all orders. (When G is finite, May, Hauschild and Waner have developed such a principle.) In the last section we indicate what one could hope for in this regard, and plan to address the actual development of a recognition principle in a future paper.