THE COMPLETE ENUMERATION OF THE 4-POLYTOPES AND 3-SPHERES WITH EIGHT VERTICES

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The enumeration of 4-polytopes and 3-spheres with eight vertices started in [1], is completed here. We show that there are precisely 624 4-polytopes and 22 non-polytopal 3-spheres with eight vertices that are not quasisimplicial nor pyramidal. We find them all and give a detailed description of the 22 non-polytopal spheres.

1. Introduction. Our purpose is to carry out a complete enumeration of the combinatorial 3-spheres and 4-polytopes with eight vertices. One part of this task has been carried out in [1], where all the quasisimplicial 3-spheres and 4-polytopes were enumerated. As mentioned in [1], the pyramidal cases are well known. Thus, to complete the enumeration, we must find all non-pyramidal non-quasisimplicial 3-spheres and 4-polytopes with eight vertices. This is done in the present work. The main result obtained here is:

THEOREM 1. *There are precisely* 624 4-*polytopes and* 22 *non-polytopal* 3-*spheres with eight vertices that are not quasisimplicial nor pyramidal.*

This theorem, combined with the results obtained in [1] and with former results mentioned in [1, §1], yields:

THEOREM 2. *There are precisely* 1294 4-*polytopes and* 42 *non-polytopal* 3-*spheres with eight vertices.*

As a by-product of the methods used for the enumeration, we get (see [6, §5.5]):

THEOREM 3. *Every* 4-*polytope with up to eight vertices is rational.* (*That is, it is combinatorially equivalent to a polytope in* R^4 *all of whose coordinates are rational.*)

The present paper should be considered a continuation of [1]. Thus the notation and terminology is that of [1]. In particular, one should