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PRODUCT FORMULAE FOR NIELSEN NUMBERS OF FIBRE MAPS

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This work simplifies proofs of a recent publication by You and gives simple sufficient conditions for Brown's product formula for the Nielsen number of a fibre map, as well as new product formulae in this context. Product formulae are also given relating absolute and relative Nielsen numbers, together with corresponding results for Reidemeister numbers.

Introduction. Let p: $E \to B$ be a fibration in which E, B and all fibres are compact connected ANR's, and let f: $E \to E$ be a fibre preserving map inducing self maps $\bar{f}$ on B and f_b on the fibre F_b over some fixed point b in the base. Since Brown [**1**] introduced his multiplicative formula $N(f) = N(f_b)N(\bar{f})$ for the Nielsen number $N(f)$ of f, various attempts have been made both to improve his results (cf. [**4**], [**5**]) and to generalize his formula (cf. [**6**], [**14**], [**18**]). In a recent paper which supercedes most of what precedes it in both of the aspects mentioned above, You [**20**] gives, among other things, necessary and sufficient conditions for Brown's formula together with a new result relating the Nielsen numbers of f, $\bar{f}$ and a relative Nielsen number $N_K(f_b)$ of f_b. Here K is the kernel of the inclusion induced homomorphism $\Pi_1 F_b \to \Pi_1 E$.

In this work we consider the second of these two results and use it (1) as a focus to give what we feel are more eccessible proofs of the results in [**20**]; (2) as a springboard to give new product theorems for fibre maps. Our results here include conditions under which $N(f) = N_k(f_b)N(\bar{f})$ and also conditions under which $N_K(f_b) = N(f_b)$. By combining these we thus obtain new sufficient conditions for Brown's formula. These conditions are simpler to verify than You's. We investigate the hypotheses of You's theorems giving conditions under which they hold. In the process we develop product formulae relating relative and absolute Nielsen numbers, together with corresponding results for Reidemeister numbers.

The unifying tool in this work is a certain exact sequence associated with a self morphism of a short exact sequence of groups. This result is a kind of non-abelian snake lemma and is a special case of a theorem (cf. [**9**]) originally proved in connection with localization of orbit sets. All the product theorems mentioned above ultimately derive from this sequence.