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ON A CLASS OF TOPOLOGICAL GROUPS MORE GENERAL THAN SIN GROUPS

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We consider a class of topological groups more general than those with small invariant nieghborhoods of the identity, SIN-groups. We refer to these more general groups as N-groups. We prove that a compactly generated N-group is a SIN-group. This result has several applications, including the following: A locally compact N-group is unimodular.

Introduction. There has been considerable interest in topological groups with small invariant neighborhoods of the identity. There is a very good bibliography of the literature on these groups in [**6**]. In this paper, we are interested in a more general class of groups which share some of the interesting properties of SIN-groups. We obtain some of these properties and attempt to determine which of the more general class are SIN-groups.

If G is a topological group and $\mathscr{B}$ is a subgroup of the group of topological automorphisms of G, we say that G is an $N(\mathscr{B})$-group or simply G is $N(\mathscr{B})$ if the following holds: For each pair of nets $\{x_\alpha\}$ in G and $\{\phi_\alpha\}$ in $\mathscr{B}$ such that $\{x_\alpha\}$ converges to the identity, the net $\{\phi_\alpha(x_\alpha)\}$ converges to the identity or fails to converge. We say that G is SIN$(\mathscr{B})$ if G has small neighborhoods of the identity which are invariant under the elements of $\mathscr{B}$. This is tantamount to saying that for any net $(\phi_\alpha, x_\alpha) \in \mathscr{B} \times G$ with $x_\alpha \to e$ we have $\phi_\alpha(x_\alpha) \to e$. If $\mathscr{B}$ is the group of all inner automorphisms of G we use "N" and "SIN" for "$N(\mathscr{B})$" and "SIN$(\mathscr{B})$" respectively.

One of our most useful results is the following: If $\mathscr{B}$ is a completely generated group of automorphisms of G in an admissible topology and G is a locally compact $N(\mathscr{B})$-group, then G is SIN$(\mathscr{B})$. We use this result to prove that a locally compact N-group is unimodular, Theorem 4; a result on invariant measures, Corollary 4; and a result on semidirect products, Proposition 3. We prove a structure theorem for locally compact totally disconnected N-groups and construct an example of a locally compact N-group which cannot be embedded in a locally compact SIN-group. This solves a problem posed in [**6**] in the category of locally compact groups. In