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ON EQUIVALENCES OF BRANCHED COVERINGS AND THEIR ACTION ON HOMOLOGY

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This paper studies equivalences of stable simple branched coverings of surfaces. We give necessary and sufficient conditions for a pair of homeomorphisms f and g of surfaces M and N respectively to be homologous to homeomorphisms $\bar{f}$ and $\bar{g}$ which form an equivalence of two prespecified stable simple branched covers ψ_1 and ψ_2. That is, homeomorphisms $\bar{f}$ and $\bar{g}$ such that

$$\begin{array}{ccc} M & \xrightarrow{\bar{f}} & M \\ \psi_1 \downarrow & & \downarrow \psi_2 \\ N & \xrightarrow{\bar{g}} & N \end{array}$$

commutes are shown to exist if and only if $\psi_{2*}f_* = g_*\psi_{1*}$ from $H_*(M)$ to $H_*(N)$.

The proof relies on a uniqueness theorem of Hamilton and Berstein, Edmonds to restate the problem in terms of self equivalences of certain simple branched covers. Many equivalences of branched covers are constructed, and it is shown that the action on homology of these equivalences generates an appropriate subgroup of the symplectic group.

CHAPTER 1

Preliminaries. All spaces and maps will be piecewise linear. M and N will denote orientable surfaces while M_g and N_g will denote closed orientable surfaces of genus g. Maps will be orientation preserving unless otherwise specified.

A *branched cover* is a finite to one open map $\varphi\colon M \to N$. In addition, we shall assume that branched covers are *primitive*, that is induce surjections on the fundamental group. The *degree* of φ is $\max_{x\in N} \#\{\varphi^{-1}(x)\}$. The *singular set* of φ is the (finite) set of points in M near which φ fails to be a local homeomorphism. The *branch set* B of φ is the image under φ of the singular set. A branched cover is *simple* if for all $x \in N$, $\#\{\varphi^{-1}(x)\} \in \{d, d-1\}$. Equivalently, a branched cover is simple if each branch point is covered by exactly one singular point, and near this point the map is of degree 2.