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ON HEREDITARILY ODD-EVEN ISOLS AND A COMPARABILITY OF SUMMANDS PROPERTY

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Our paper contains three theorems on regressive isols that are hereditarily odd-even. Two are characterizations of hereditarily odd-even isols in terms of a parity property of the isol and a property on the comparability of summands of the isol. In the third theorem, we show that if a regressive isol has a special comparability of summands property, then it has a predecessor that is hereditarily odd-even.

1. Introduction. The results presented in the paper developed from an interest in regressive isols that are hereditarily odd-even and in a special property about the comparability of summands that such isols are known to possess. The term *hereditarily odd-even* isol was introduced by T. G. McLaughlin in [**3**]. These are isols that are infinite, and each predecessor of the isol is either even or odd. It is among the regressive isols that these isols are especially interesting, for in that setting it is known that the hereditarily odd-even isols are the same as the hyper-torre isols (cf. [**3**]). E. Ellentuck studied hyper-torre isols in [**2**], and by using them it was shown that certain natural collections in the isols are models of the universal properties of arithmetic.

In this paper we are interested in regressive isols. We shall assume that the reader is familiar with topics in the monograph [**3**] on regressive sets and the theory of isols. In particular we use the metatheorem of A. Nerode that states that universal Horn sentences which are true in ω extend to statements which are true in the isols. This result is discussed in [**3**, Chapter 12]. The main concepts that we need are contained in the following two definitions.

DEFINITION D1. An isol is said to have *parity* if it is even or odd. An isol is said to have *4-parity* if it can expressed in one of the forms $4y$, $4y + 1$, $4y + 2$, or $4y + 3$.

DEFINITION D2. An infinite regressive isol Y is said to have *comparability of summands* if whenever $Y = A + B$, either $A \leqq {}^* B$ or $B \leqq {}^* A$. Y