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ON JUNG'S CONSTANT AND RELATED CONSTANTS IN NORMED LINEAR SPACES

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In this paper several results on certain constants related to the notion of Chebyshev radius are obtained. It is shown in the first part that the Jung constant of a finite-codimensional subspace of a space $C(T)$ is 2, where T is a compact Hausdorff space which is not extremally disconnected. Several consequences are stated, e.g. the fact that every linear projection from a space $C(T)$, T a perfect compact Hausdorff space, onto a finite-codimensional proper subspace has norm at least 2.

The second discusses mainly the "self-Jung constant" which measures "uniform normal structure." It is shown that this constant, unlike Jung's constant, is essentially determined by the finite subsets of the space.

1. Jung constant in $C(T)$ spaces.

For a bounded subset A of a normed linear space E and a subset Y of E we denote by diam A the diameter of A ($\sup_{x,y\in A}\|x-y\|$), by $r_Y(A)$ the relative Chebyshev radius of A with respect to Y ($\inf_{y\in Y}\sup_{x\in A}\|x-y\|$), and by $Z_Y(A)$ the relative Chebyshev center set of A in Y ($\{y\in Y; \sup_{x\in A}\|x-y\| = r_Y(A)\}$). The Jung constant of E is $J(E)=\sup\{2r_E(A); A\subset E, \operatorname{diam} A = 1\}$. It is easily seen that $1 \leqq J(E) \leqq 2$. For n-dimensional spaces E_n, it was shown by Jung [12] that $J(l_2^n) = (2n/(n+1))^{1/2}$ and $J(E_n) = 1$ if and only if $E_n = l_\infty^n$. Bohnenblust [2] showed that $J(E_n) \leqq 2n/(n+1)$, and Leichtweiss [14] characterized the extremal case (in the 2-dimensional case it is the hexagonal plane). In the infinite-dimensional case, it was shown that $J(l_2) = \sqrt{2}$ (Routledge [20]), and that $J(E) = 1$ if and only if $E = C(T)$ for a Stonian T, i.e. if $E \in \mathscr{P}_1$ (Davis [5]) (cf. also [10], pages 91–92 in [11] and §6 in [4]).

Studying intersections of balls with subspaces, Franchetti [6] deduced that for every finite-codimensional subspace E of $C[a, b]$ we have $J(E) \geqq 3/2$. A stronger and more general result is true.

1.1. PROPOSITION. *If the compact Hausdorff space T is not extremally disconnected, then for every finite-codimensional subspace E of $C(T)$ we have $J(E) = 2$.*