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# IDEAL MATRICES. III

OLGA TAUSSKY

*Dedicated to the memory of Ernst G. Straus*

**In this paper ideal matrices with respect to ideals in the maximal order of an algebraic number field are connected with the different of the field and with group matrices in the case of normal fields whose maximal order has a normal basis.**

**1.** Let $F$ be an algebraic number field of degree $n$ over $Q$. Let $\mathfrak{O}$ be the maximal order of $F$ and $\mathfrak{a}$ an ideal in $\mathfrak{O}$. Let $\omega_1,\ldots,\omega_n$ be a $Z$-basis of $\mathfrak{O}$ and $\alpha_1,\ldots,\alpha_n$ a $Z$-basis of $\mathfrak{a}$. Then there exists an integral $n \times n$ matrix $A$ such that

$$A\begin{pmatrix}\omega_1\\ \vdots\\ \omega_n\end{pmatrix} = \begin{pmatrix}\alpha_1\\ \vdots\\ \alpha_n\end{pmatrix}. \tag{1}$$

$A$ is called an ideal matrix for $\mathfrak{a}$. The ideal defines a representation module for $\mathfrak{O}$. A change of bases replaces $A$ by $UAV$ where $U$, $V$ are unimodular $n \times n$ $Z$-matrices and conversely, to any $U$, $V$ there exist bases for $\mathfrak{O}$ and $\mathfrak{a}$ such that $UAV$ is an ideal matrix for $\mathfrak{a}$.

**2.** THEOREM 1. *Let* $(j)$ *denote the family of embeddings of* $F$ *in the complex numbers. Then, suppressing the superscript* (1),

$$A\begin{pmatrix}\omega_1 & \omega_1^{(2)} & \cdots & \omega_1^{(n)}\\ \vdots & \vdots & & \vdots\\ \omega_n & \omega_n^{(2)} & \cdots & \omega_n^{(n)}\end{pmatrix} = \begin{pmatrix}\alpha_1 & \alpha_1^{(2)} & \cdots & \alpha_1^{(n)}\\ \vdots & \vdots & & \vdots\\ \alpha_n & \alpha_n^{(2)} & \cdots & \alpha_n^{(n)}\end{pmatrix}. \tag{2}$$

Hence we have

THEOREM 2. $A = (\alpha_i^{(j)})(\omega_i^{(j)})^{-1}$.

Both theorems have trivial proofs. Let $\alpha$ be any integral generator of $F$. Then there exists an integral matrix $X$ such that $\det(\omega_i^{(j)})\det X = \pm$ different $\alpha$. Further, $(\omega_i^{(j)})(\omega_i^{(j)})'$ is the so-called discriminant or trace matrix of $\mathfrak{O}$ where $'$ stands for the transpose and the determinant of the above matrix product is the discriminant of the maximal order.