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UTTERLY INTEGER VALUED ENTIRE FUNCTIONS (I)

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In Loving Memory of Ernst G. Straus

An entire function $f(z)$ is called *utterly integer valued* if $f(x)$ and all its derivatives assume integer values for all integer z. A historical survey of the theory of such functions is given, and a new class of them is constructed. There is no utterly integer valued entire functions of finite order except polynomials.

1. Introduction. G. Pólya [**21–23**] studied entire functions which take integral values at all nonnegative integral points. This is generally considered to be the origin of the research on arithmetic properties of analytic functions. Unless otherwise stated, a "function" in this paper means an analytic (entire) functin of a complex variable.

A function $w = f(z)$ is called *an integer valued function* if $f(l) =$ integer for $l = 0, 1, 2, \ldots$ [**21**]. A function $f(z)$ is called a *completely* integer valued function if $f(l) =$ integer for $l = 0, \pm 1, \pm 2, \ldots$ [**9**, **23**]. A function $f(z)$ is called a *q-fold integer valued function* if $f(z)$ and its derivatives $f'(z), f''(z), \ldots, f^{(q-1)}(z)$ are all integer valued [**28**, **30**]. On the other hand, a *Hurwitz function* is defined to be a function $f(z)$ which together with *all* its derivatives assumes integral values at the origin $z = 0$, [**32–35**]. If the function $f(z)$ and all its derivatives assume integral values at k consecutive integral points, say $z = 0, 1, 2, \ldots, k - 1$, then $f(z)$ is called a *k-point Hurwitz function* [**36–37**, **40–44**]. These concepts of integral valued functions lead to the following

DEFINITION. Given two sets S and T of complex numbers, we say that a function $f(z)$ is *infinitely T-valued at S*, if the function and all its derivatives assume values in T at all points of the set S.

Here S is called *the set of interpolation* and T is called *the value set*. An infinitely T-valued function is called *an utterly T-valued function* if the set of interpolation S is *unbounded*. In particular, if S and T are the set of all rational integers, or the sets of all algebraic integers in a fixed imaginary quadratic number field K, then we say that an utterly T-valued function at S is an *utterly K-integer valued function*.