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PRIMES OF THE FORM $[n^c]$

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Dedicated to the memory of Ernst G. Straus

Methods of Vinogradov for estimating exponential sums over primes are modified and made easier to use. Comparisons are made with approaches of Heath-Brown and Vaughan.

1. Introduction. In 1939 I. M. Vinogradov developed a method of estimating exponential sums over primes. His method reduces the estimation of a sum $S = \sum_{p \le N} F(p)$ to the estimation of sums of type 1,

$$\sum_{X \le x \le 2X} a(x) \sum_{\substack{Y < y \le Y_1 \\ xy \le N}} F(xy),$$

where $Y_1 \le 2Y$, Y is large, and sums of type 2,

$$\sum_{X \le x \le 2X} a(x) \sum_{\substack{Y \le y \le 2Y \\ xy \le N}} b(y) F(xy),$$

where X and Y are large.

R. C. Vaughan proved an identity which allows one to express S as the sum of type 1 and type 2 sums:

$$\sum_{V \le n \le X} \Lambda(n) F(n) = S_1 - S_2 - S_3,$$

where

$$S_1 = \sum_{d \le U} \sum_{k \le X/d} \mu(d) \log k F(dk);$$

$$S_2 = \sum_{k \le UV} a(k) \sum_{r \le X/k} F(kr),$$

with

$$a(k) = \sum_{\substack{d \le U,\, n \le V \\ dn = k}} \mu(d) \Lambda(n);$$

and

$$S_3 = \sum_{m > U} \sum_{V \le n \le X/n} \Lambda(n) \left(\sum_{\substack{d \mid m \\ d \le U}} \mu(d) \right) F(mn),$$