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STABLE AUGMENTATION QUOTIENTS OF ABELIAN GROUPS

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To the Memory of Ernst Straus

Let G be a finite abelian p-group, $\mathbf{Z}G$ the associated integral group ring, and Δ its augmentation ideal. This paper determines the stable structure of the augmentation quotients Δ^n/Δ^{n+1} and the structure of the graded ring $\operatorname{gr}\mathbf{Z}G$. It also gives an application to the dimension subgroup problem, extending earlier results of Gupta-Hales-Passi.

1. Introduction. Let $\mathbf{Z}G$ be the integral group ring of a finite abelian group G. Denote by Δ the augmentation ideal of $\mathbf{Z}G$, i.e. the kernel of the map from $\mathbf{Z}G$ to $\mathbf{Z}$ sending each group element to 1. Further denote by Q_n the nth "augmentation quotient" Δ^n/Δ^{n+1}. Then Bachman and Grunenfelder [1] have shown that, for all $n \geq n_0 = n_0(G)$, we have $Q_n \cong Q_{n+1} \cong Q_{n+2} \cdots$ as abelian groups. Let $Q_\infty = Q_\infty(G)$ denote the "eventual" isomorphism type of the Q_n. A number of papers ([2], [5], [6], [7], [10], [11], [12], [13], [15]) have been devoted to the determination of $Q_\infty(G)$ in terms of G. In [4] we gave a conjecture for the structure of $Q_\infty(G)$ and verified this conjecture whenever $G \cong (C_{p^n})^m$ for some m and n. Here we shall establish the truth of this conjecture for all finite abelian G, and in the process determine $n_0 = n_0(G)$ and the structure of the graded ring $\operatorname{gr}\mathbf{Z}G$ associated to $\mathbf{Z}G$. We also give an application (extending a result in [3]) to the dimension subgroup problem.

The reader should consult Passi [8] for general background on the subject, and [4] for more specific background on this problem.

2. Description of results. Without loss of generality we may assume that G is a finite abelian p-group, in which case Q_∞ is also easily seen to be such a group. One way of viewing our problem is that we wish to determine the invariants of Q_∞ in terms of those of G. Instead, however, we give an explicit presentation of (a group isomorphic to) Q_∞ from which the invariants of Q_∞ can be determined in a straightforward (though tedious) manner.

Define an abelian group Q_G via generators and relations as follows: let P_G denote the poset of cyclic subgroups H of G. (So P_G is a tree with