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LOWER BOUNDS ON BLOCKING SETS

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In memory of Ernst Straus

In a geometry the notion of *blocking set* ordinarily refers to a set of points meeting every line. (For projective planes convention requires in addition that the blocking set not contain all the points of any one line.) In this paper we obtain lower bounds for the size of a blocking set for projective planes and inversive planes, which are equal to or improvements on the best previously known bounds. If the notion of blocking set is generalized to families of disjoint subspaces rather than sets of points, then (partial) spreads are included, and we obtain a lower bound for the size of a maximum partial spread of m-spaces in projective $(2m + 1)$-space. The technique is that of Glynn, counting various sets of intersecting subspaces in two ways to obtain inequalities.

If we are considering any geometry (affine, projective, inverse, etc...) we can use the following definition:

DEFINITION. A set S of mutually disjoint k-subspaces is a (k, l)-*blocking set* if every l-subspace meets some member of S.

In the case of $k = 0$, $l = 1$ we have ordinary blocking sets, except for the case of the projective plane, where conventionally the condition that S contain no line is required for blocking sets. If $k = l$, then we have a *maximum partial spread* of k-spaces. These are the two cases for which there are some known bounds.

We consider first the case of $k = 0$, $l = 1$ in the inversive plane of order q [**4**]. The relevant features of this geometry are that there are $q^2 + 1$ points, $(q^2 + 1)q$ lines (called "circles" sometimes), $q(q + 1)$ lines through each point, $(q + 1)$ lines through any two points, and one line through any three points. Suppose we have a set S of x points, and denote by n_i the number of lines containing exactly i points of S, $i \geq 0$. (We note that $n_i = 0$ for $i > q + 1$ as lines have $q + 1$ points.) If S is a blocking set, then $n_0 = 0$.

Counting all lines gives

$$(1) \qquad \sum_{i \geq 1} n_i = (q^2 + 1)q.$$