

## LATTICES OF COMPLETELY REGULAR SEMIGROUP VARIETIES

F. J. PASTIJN AND P. G. TROTTER

Let  $\rho$  be a fully invariant congruence on the free completely regular semigroup  $F_X^{\text{CR}}$  of countably infinite rank. Let  $\rho_{\min}$  and  $\rho^{\min}$  be the least congruences on  $F_X^{\text{CR}}$  with respectively the same trace and the same kernel as  $\rho$ . Let  $\rho_{\max}$  and  $\rho^{\max}$  be the greatest congruences on  $F_X^{\text{CR}}$  with respectively the same trace and the same kernel as  $\rho$ . These congruences are shown to be fully invariant. We construct a network of varieties corresponding to the congruences  $\dots, \rho^{\max}, \rho_{\max}, \rho, \rho^{\min}, \rho_{\min}, (\rho_{\min})^{\min}, (\rho^{\min})_{\min}, \dots$  and their intersections. Intervals between successive joins of the network, in the lattice of subvarieties of completely regular semigroups, are characterised as direct products of particular subintervals. By comparing the network with the chain of varieties that are each generated by a free completely regular semigroup of finite rank we get information on the network and the chain.

**1. Introduction.** *Completely regular semigroups* are semigroups that are unions of their subgroups. They may be regarded as universal algebras with an associative binary operation (multiplication) and a unary operation (inversion). As universal algebras, completely regular semigroups form a variety determined by the identities

$$(1) \quad x = xx^{-1}x, \quad xx^{-1} = x^{-1}x, \quad (x^{-1})^{-1} = x.$$

Let  $\mathbf{CR}$  denote this variety and  $L_{\mathbf{CR}}$  denote the lattice of subvarieties of  $\mathbf{CR}$ .

Descriptions of sublattices of  $L_{\mathbf{CR}}$  have proliferated in recent years. The sublattices for which detailed results have been obtained are all bounded above by  $\mathbf{BG} \vee \mathbf{O}$  where  $\mathbf{BG}$  denotes the variety of all bands of groups and  $\mathbf{O}$  denotes the variety of all orthodox completely regular semigroups. We will see that these sublattices lie at the bottom of  $L_{\mathbf{CR}}$ . Varieties under  $\mathbf{BG} \vee \mathbf{O}$  include the variety  $\mathbf{G}$  of all groups, the variety  $\mathbf{B}$  of all bands and the variety  $\mathbf{CS}$  of all completely simple semigroups. The lattice of subvarieties of  $\mathbf{B}$  has been fully described in [1], [6] and [7]. The lattice of subvarieties of  $\mathbf{CS}$  has been studied in [17], [18], [21], [28] and [31] while other sublattices of  $L_{\mathbf{CR}}$  have been investigated in [15], [19], [24], [25], [27], [30] and [22].