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FREE PRODUCTS OF TOPOLOGICAL GROUPS WITH AMALGAMATION

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It is proved that the free product of any two k_ω-groups with a compact subgroup amalgamated is a k_ω-group, and in particular, Hausdorff.

1. Introduction. In recent years much work has been done on describing the topology of free products of topological groups (see for example [**1, 3, 8, 10, 12, 13, 15**]). From there it is natural to progress to free products with amalgamation.

One would hope that the free product with amalgamation of any Hausdorff topological groups exists, is Hausdorff and its underlying group structure is the amalgamated free product of the underlying groups. This would include as a special case Graev's theorem [**2**] that the free product of Hausdorff groups is Hausdorff. As his proof is certainly non-trivial, it should not be expected that this "hope" will be easily verified, even if the result is true.

The first contribution to this problem was by Ordman [**13**], who showed that the amalgamated free product of certain locally invariant Hausdorff topological groups is Hausdorff. The next contribution was by Khan and Morris [**5**] who proved the Hausdorffness of the free product of Hausdorff groups with a central subgroup amalgamated. This has recently been extended by Katz and Morris [**4**] to free products of k_ω-groups with a closed normal subgroup amalgamated.

Most of the work on free topological groups and free products of topological groups in fact deals with topological groups which are k_ω-spaces. Therefore, the result we would like to have is that the amalgamated free product of k_ω-groups is a k_ω-group. This would imply La Martin's theorem that epics in the category of k_ω-groups have dense range. (See [**6**], [**11**] and [**14**].) We prove here that the free product of any two k_ω-groups with a compact subgroup amalgamated is a k_ω-group and in particular Hausdorff. This result includes a large class of examples not covered by [**4**] or [**13**], since every connected locally compact Hausdorff topological group, G is a k_ω-group and has a compact subgroup K such