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CLOPEN REALCOMPACTIFICATION OF A MAPPING

TAKESI ISIWATA

In this note, we give a necessary and sufficient condition on φ: $X \to Y$ for $\upsilon\varphi$ to be an open perfect mapping of υX onto υY and other related results.

Throughout this paper, by a space we mean a completely regular Hausdorff space and mappings are continuous and we assume familiarity with [1] whose notation and terminology will be used throughout. We denote by φ: $X \to Y$ a map of X onto Y, by βX (υX) the Stone-Čech compactification (Hewitt realcompactification) of X and by $\beta\varphi$ ($\upsilon\varphi = (\beta\varphi)|\upsilon X$) the Stone extension (realcompactification) over βX (υX) of φ.

Concerning clopenness of $\upsilon\varphi$ of a clopen map φ: $X \to Y$ the following results are known.

THEOREM A (*Ishii* [4]). *If φ: $X \to Y$ is an open quasi-perfect map, then $\upsilon\varphi$ is an open perfect map of υX onto υY.*

THEOREM B (*Morita* [8]). *If φ: $X \to Y$ is a clopen map such that the boundary of each fiber is relatively pseudocompact, then $\upsilon\varphi$ is also a clopen map of υX onto υY.*

In §2, concerning Theorem A we give a necessary and sufficient condition on φ for $\upsilon\varphi$ to be an open perfect map of υX onto υY without using the theory of hyper-spaces (Theorem 2.3 below) and a necessary and sufficient condition on φ for $\upsilon\varphi$ to be an open RC-preserving map of υX onto υY under some condition (Theorem 2.6 below).

We use the following notation and abbreviation: $C(X)$ is the set of real-valued continuous functions defined on X, $C(X; \varphi) = \{f \in C(X); f$ is φ-bounded$\}$, Bd A = the boundary of A, usc = upper semicontinuous, lsc = lower semicontinuous and ω (ω_1) = the first infinite (uncountabel) ordinal, clopen = closed and open.

1. Definitions and Lemmas.

1.1. DEFINITION. Let φ: $X \to Y$. $f \in C(X)$ is said to be *φ-bounded* if $\sup\{|f(x)|; x \in \varphi^{-1}(y)\} < \infty$ for every $y \in Y$. Whenever f is φ-bounded,