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G-BORDISM WITH SINGULARITIES AND G-HOMOLOGY

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The bordism and cobordism theories of singular G-manifolds of specified kinds are used to represent various ordinary G-homology and cohomology theories, and their relationship to each other, as well as their relationship to non-singular G-bordism, is studied.

1. Introduction. Sullivan once pointed out that ordinary homology may be viewed geometrically as a bordism theory with singularities. This has been formally established by Baas in [**1**] and by Buoncristiano, Rourke and Sanderson in [**3**]. Dually, the associated cobordism theories represent ordinary cohomology.

Let G be a finite group. One then has several notions of what is meant by ordinary G-cohomology. The first to be proposed was the functor $X \mapsto H^*(X \times_G EG)$ for a G-space X, where EG denotes the universal contractible free G-space. Subsequently, Bredon [**2**] and Illman [**6**] described a theory of the following type. Let $\mathcal{G}$ denote the category whose objects are the G-spaces G/H for subgroups H and whose morphisms $G/H \to G/K$ are the G-equivariant maps. A contravariant coefficient system is then a contravariant functor T from $\mathcal{G}$ to the category of abelian groups. The associated ordinary G-cohomology theory is a generalized G-cohomology theory (see [**2**]) with dimension axiom of the form

$$H^n_G(G/K; T) = \begin{cases} T(G/K) & \text{if } n = 0, \\ 0 & \text{if } n \neq 0. \end{cases}$$

More recently, it has been shown, [**18**], [**8**], [**9**], that this theory extends to an $RO(G)$-graded theory when the coefficient system T extends to a Mackey functor (in that it admits a transfer). In this theory, the coefficient system A: $G/H \mapsto A(H)$, the Burnside ring of H, then assumes the role played by Z-coefficients nonequivariantly. As yet, no geometric description of cycles in the non-integrally graded part of the dual theory, $H^G_*(X)$ exists; if V is a non-trivial G-module, how does one view the classes in $H^G_V(X; A)$?

It is not clear how to extend Sullivan's ideas to represent these G-cohomology theories as singular cobordism theories. Moreover, two